

Weighted Hardy spaces, the uncertainty principle for Fourier transforms, and the existence of translation invariant subspaces

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Abstract

In the first part of the course we associate with certain weight functions on the real line and on the positive axis several classes of weighted Hardy spaces and some Banach spaces of entire functions, which can be identified with closed subspaces of the corresponding Lebesgue spaces. We show that some of these spaces can be characterized by the rate of decay of the Fourier transforms of their elements at infinity. We thereby obtain in a general setting extensions of the classical Paley-Wiener theorems and sharp forms of the uncertainty principle for Fourier transforms which imply most of the known results in this area. We also show that some of these Banach spaces of entire functions are invariant under differentiation, and use this fact to prove that the integers as well as the nonnegative integers are sampling sets for these spaces. In the second part of the course we consider the problem of existence of proper translation invariant subspaces of weighted L^2 spaces on the integer group Z , which is open for general weights, and is equivalent to the hyperinvariant subspace problem for invertible weighted bilateral shifts on Hilbert space. Using the above mentioned sampling theorems we obtain a positive answer for a large class of weights. If time permits, we shall also show by different methods that the answer is positive for all even weights.