Dilation operator radius and applications

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Recall that an operator T acting on a Hilbert space H admits a *unitary ρ-dilation* ($\rho > 0$), in the sense of B. Sz. Nagy and C. Foias, if there exists a Hilbert space H containing H as a closed subspace and a unitary operator U on H such that

$$T^n = \rho P_H U^n \mid H \qquad (n \geq 1),$$

where P_H is the orthogonal projection from H onto H. We denote by $C_\rho(H)$ the set of all operators on H which admit a unitary ρ-dilation and we write $C_\infty(H) = \bigcup_{\rho>0} C_\rho(H)$. A famous result of B. Sz. Nagy tells us that $C_1(H)$ consists of all contractions on H and C. A. Berger proved in a paper entitled "*A strange dilation theorem*" that $C_2(H)$ consists of all operators T acting on H with numerical radius less or equal to one. The study of operators belonging to $C_\infty(H)$ was elarged upon by many authors in several directions.

For any $T \in C_\infty(H)$, we can define the following stricly positive number

$$\rho(T) = \inf\left\{\rho > 0 : \rho \in R_+^* \text{ and } T \in C_\rho(H)\right\}.$$

The number $\rho(T)$ is the smallest $\rho > 0$ such that $T \in C_\rho(H)$ and will be called the *dilation operator radius* of T. We establish several properties of this radius. This study enables us to give results in several directions and to retrieve in a short way recent results which are concerned with the Aluthge transformation.