Operator Theory and Complex Geometry

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Abstract

We will discuss the relation of Hilbert spaces of holomorphic functions which are modules over the polynomials in m variables with Hermitian holomorphic vector bundles, the classification of these Hilbert modules, and the application of this approach to several concrete problems in operator theory. Although we will couch the concepts and definitions in the context of several complex variables, the emphasis will be one the one variable case and the classical examples of the Hardy and weighted Bergman spaces on the unit disk. Also, while complex geometry will be used as a framework for the techniques introduced, every effort will be made to express the proofs and definitions in the language of operator theory. Some open problems will be discussed.

Lecture 1: Introduction to quasi-free Hilbert modules - Examples of such modules for the disk, ball and polydisk, their properties and approaches to their classification.
Lecture 2: Reducing submodules of quasi-free Hilbert modules - Characterizations of reducing submodules and their determination for some examples including powers of the Bergman shift.
Lecture 3: Isomorphic submodules - Submodules of a quasi-free Hilbert module isomorphic to it will be shown to be extremely rare.