Coulomb Gas in 2D

Nikolai G. Makarov

Abstract

I will survey various aspects of the theory of random normal matrices, a statistical Coulomb gas model well known in the physical and mathematical literature. The theory has three closely related and equally exciting levels – classical, statistical, and field theoretical. Accordingly, my plan for the mini-course has three parts:

(a) classical limit: potential theory, quadrature domains, Laplacian growth;

(b) large N expansion: Gaussian fluctuations, Polyakov-Alvarez formula, double scaling;

(c) conformal field theory of Coulomb gas: Ward's identities, stress-energy tensor, and Schramm's SLEs.

In the lectures, which are meant to be 100% mathematical (Complex Analysis), I will follow a series of theoretical physics papers by P. Wiegmann, A. Zabrodin, et al. , and my own work with Y. Ameur, H. Hedenmalm, and with N. G. Kang.