Completeness of families of exponential functions in L^2-spaces

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Abstract

Consider a family of exponential functions $e^{i\lambda x}$ where the frequency λ belongs to a fixed subset Λ of the complex plane. The question of completeness (density of finite linear combinations) of such families in spaces $L^p(\mu)$, where μ is a positive finite measure on the real line, is one of the classical questions of Harmonic Analysis.

If $p = 2$, μ is Lebesgue measure on an interval and Λ is a sequence, the problem becomes the famous Beurling-Malliavin problem that was solved in the early sixties. Its solution is considered to be one of the deepest results in the 20th century Harmonic Analysis.

The case when $p = 2$, μ is a finite positive measure on $\mathbb{R}$ and Λ is an interval, is another well-known problem that was discussed by Wiener and later posted by Kolmogorov and Krein in the fifties. Until recently, this problem stood completely open even for simple choices of μ.

For $p = 1$ and μ, Λ the same as above, one obtains the so-called Gap Problem that stems from the well-known Beurling Gap Theorem proved in the late fifties. A solution to this problem is also a recent result.

In my talks I will discuss a modern approach for this set of problems, their relations with adjacent fields, such as spectral problems for differential operators and Bernstein's weighted approximation, latest results and open questions.