Schroeder's Equation in Several Variables

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Abstract

In one variable, Schroeder's functional equation

$$f \circ \varphi = \lambda f$$

was solved by Koenigs in 1884 when f and φ are analytic in a neighborhood of an attractive fixed point of φ. In particular, Koenigs showed that solutions are possible only when $\lambda = \varphi'(0) \neq 0$ and, for each of these values of λ, there is a one-dimensional solution space. This result and its extensions have provided a means of understanding the iteration of an analytic function near fixed points as well as providing a foundation for the classification and study of composition operators on spaces of analytic functions on the unit disk. The goal of this research is to extend Koenigs' work in order to classify and better understand composition operators on spaces of analytic functions in several variables.

We will review earlier work on Schroeder's equation in several variables, for example, in the case when φ is a map of the unit ball into itself and 0 is an attractive fixed point of φ. In this case, we take Schroeder's equation to be

$$f \circ \varphi = \varphi'(0) f$$

where f is an analytic map of the unit ball in $\mathbb{C}^N$ into $\mathbb{C}^N$ for which $f'(0)$ is invertible. We will begin with work from 2003 of Barbara MacCluer and the speaker and a description of how 'resonance' is sometimes an obstruction to the existence of solutions. We will conclude with more recent results of Ruth Enoch, Robert Bridges, Maria Neophytou, and the speaker on the (occasional) existence of solutions even in the case of resonance.