Linear dynamics and ergodic theory

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Abstract

As everybody knows, ergodic theory studies the iterates of a measurable map $T : (X, \mu) \to (X, \mu)$ defined on a probability space (X, μ). Usually, the measure μ is assumed to be T-invariant and the transformation T is ergodic or mixing (in the weak or the strong sense). On the other hand, linear dynamics is the study of the iterates of continuous linear operators defined on a topological vector space. Here, the main characters are the so-called *hypercyclic* operators; i. e. the operators having at least one dense orbit.

It is easy to see that if $T : X \to X$ is a bounded linear operator on a Banach space X and if there is a T-invariant Borel probability measure μ on X such that each non-empty open set has positive measure and T is ergodic with respect to μ, then T is hypercyclic. Hence, it is natural to look for conditions ensuring that such a measure does exist. This problem goes back to a 1995 paper by E. Flytzanis, and it has been intensively studied in the last 6 years, mostly by F. Bayart and S. Grivaux. The result is a complete characterization of linear operators which happen to be *weakly mixing* with respect to some nontrivial Gaussian measure. In this series of lectures, I plan to explain in detail this characterization.