

OPERATOR-VALUED FOURIER MULTIPLIERS AND ONE-PARAMETER SEMIGROUPS

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In these lectures we will give an introduction to a subject of semigroup theory which obtained much attention in recent years, the problem of maximal regularity. Given a one-parameter semigroup $(T(t))_{t \geq 0}$ with generator A and $f \in L^p(0, \tau; X)$, the inhomogeneous Cauchy problem

$$u'(t) = Au(t) + f(t), \quad u(0) = 0$$

has a unique mild solution $u = T * f$. A fundamental question is whether this solution enjoys "maximal regularity", i.e. if $u' \in L^p(0, \tau; X)$. A necessary condition is that the semigroup is holomorphic. Holomorphy is also sufficient whenever the underlying space is a Hilbert space. By a famous result by Kalton and Lancien from 2000 the converse is also true: the maximal regularity property for all generators of a holomorphic semigroup even implies that the underlying space is a Hilbert space (if mild conditions on the space are satisfied). In order to describe maximal regularity on spaces other than Hilbert spaces, operator-valued Fourier multipliers play an essential role. We will establish the corresponding Michlin- and Marcinkiewicz type of results which lead to a characterization of the maximal regularity property. There are several interesting operator theoretical questions (such as functional calculus, multipliers with respect to Schauder bases) coming into the play, which we discuss. We also demonstrate why the question of maximal regularity is important for the treatment of non-linear problems. These lectures are intended to give an easy introduction, but further developments will be indicated.

References

- (1) Arendt, W. *Semigroups and Evolution Equations: Functional Calculus, Regularity and Kernel estimates*. HANDBOOK OF DIFFERENTIAL EQUATIONS. Evolutionary Equations, **Vol. 1**. C.M. Dafermos and E. Feireisl eds., Elsevier (2004), 1–86.
- (2) W. Arendt, S. Bu: *Fourier series in Banach spaces and maximal regularity*. In: Vector measures, integration and related topics. Oper. Theory Adv. Appl., Birkhäuser, Basel 201 (2010), 21-39.

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