Modern approaches to the invariant subspace problem

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Abstract

There is an outstanding problem in operator theory, the so-called "Invariant Subspace Problem", which has been open for more than a half of century. There have been significant achievements on occasions, sometimes after an interval of more than decade, but its solution seems nowhere in sight. The invariant subspace problem for a complex Banach space X of dimension at least 2 is the question whether every bounded linear operator $T : X \to X$ has a nontrivial closed T-invariant subspace (a closed linear subspace M of X which is different from $\{0\}$ and X such that $T(M) \subset M$).

For the most important case of Hilbert spaces H the problem is still open, though Enflo and Read showed that the invariant subspace problem is false for some Banach spaces.

There have been many significant developments in this branch of operator theory. Therefore, it was necessary to be selective in our choice of material. Some themes to be discussed include the following:

Lecture 1:
Functional calculus for Beurling algebras, the Atzmon–Wermer results. A more transparent presentation of Davie's work on Bishop operators (and his own functional calculus), and subsequent developments.

Lecture 2:
Universal operators. The study of composition operators and their invariant subspaces, as key examples.

Lecture 3:
Compact operators and their generalizations, namely, finitely strictly singular operators and strictly singular operators.

Most of the lectures are based on the book "Modern approaches to the invariant subspace problem" by I. Chalendar and J.R. Partington, Cambridge University Press, 2011.