The Boundedness and compactness of a class of \( h \)-Fourier integral operators

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Abstract

For \( \varphi \in \mathcal{S}(\mathbb{R}^n) \) (the Schwartz space), the integral operators

\[
F_h \varphi(x) = \int \int e^{i\frac{1}{h}(S(x,\theta) - y\theta)} a(x, \theta) \varphi(y) \, dy \, d\theta
\]

appear naturally in the expression of the solutions of the semiclassical hyperbolic partial differential equations and in the expression of the \( C^\infty \)-solution of the associate Cauchy’s problem. Which appear two \( C^\infty \)-functions, the phase function \( \phi(x, y, \theta) = S(x, \theta) - y\theta \) and the amplitude \( a \).

In this work, we apply the technique of [1] to establish the boundedness and the compactness of the operators (1). To this end we give a brief and simple proof for a result of [1] in our framework.

We mainly prove the continuity of the operator \( F_h \) on \( L^2(\mathbb{R}^n) \) when the weight of the amplitude \( a \) is bounded. Moreover, \( F_h \) is compact on \( L^2(\mathbb{R}^n) \) if this weight tends to zero. Using the estimate given in [4] for \( h \)-pseudodifferential (\( h \)-admissible) operators, we also establish an \( L^2 \)-estimate of \( \| F_h \| \).

We note that if the amplitude \( a \) is just bounded, the Fourier integral operator \( F_h \) is not necessarily bounded on \( L^2(\mathbb{R}^n) \). Recently, M. Hasanov [2] and B. Messirdi-A. Senoussaoui [3] constructed a class of unbounded Fourier integral operators with an amplitude in the Hörmander’s class \( S_{1,1}^0 \) and in \( \bigcap_{0 < \rho < 1} S_{\rho,1}^0 \).

Keywords: \( h \)-Fourier integral operators, \( h \)-pseudodifferential operators, symbol and phase.

References


[3] B. Messirdi and A. Senoussaoui, A class of unbounded Fourier integral operators with symbol in \( \bigcap_{0 < \rho < 1} S_{\rho,1}^0 \), International Journal of Mathematical Analysis, Vol 1, no. 18, (2007), 851-860.