SHIFT-INVARIANT SUBSPACES AND INPUT/STATE/OUTPUT LINEAR SYSTEMS: THE WEIGHTED BERGMAN SPACE SETTING

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It is well known that subspaces of the Hardy space over the unit disk which are invariant under the backward shift occur as the image of an observability operator associated with a discrete-time state/output linear system with stable state-dynamics, as well as the functional-model space for a Hilbert space contraction operator, while forward shift-invariant subspaces have a representation in terms of an inner function (Beurling's theorem) which can be presented in terms of an explicit transfer-function realization, and there is a well developed theory of interpolation (Nevanlinna-Pick interpolation theory) for contractive multipliers between vector-valued Hardy spaces. Here we discuss these issues in the context of weighted Bergman spaces on the unit disk. Specifically, subspaces invariant for the adjoint of the Bergman shift can be represented as the range of a observability operator associated with a higher-order discrete-time state/output linear system, there is a Bergman analogue of the canonical functional-model space of de Branges-Rovnyak which serves as the model space for n-hypercontractive Hilbert space operators, the analogue of Beurling's theorem for the representation of a forward Bergman-shift invariant subspace splits into three distinct forms, and there are some interpolation results of Nevanlinna-Pick type at least for contractive multipliers from a Hardy space to a weighted Bergman space. There are also extensions of these ideas to multivariable weighted Bergman spaces in both commutative and noncommutative settings. These results provide a further elaboration of those described by Anders Olofsson at the 2011 CAOT. This series of lectures describes ongoing joint work of the speaker with Vladimir Bolotnikov of the College of William and Mary.