Asymptotic behaviour of a singularly perturbed convection-diffusion problem in a rectangle with discontinuous boundary data at the corners

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Resumen

In former works [1, 2, 3], we have studied the asymptotic behaviour of the solution of several singular perturbation convection-diffusion problems with discontinuous data defined on different unbounded domains (quarter plane, an infinite and a semi-infinite strip, a sector). In this work [4], we analyze a problem of the same type but defined on a bounded domain, more interesting for practical purposes. We consider a singularly perturbed convection-diffusion equation, $-\varepsilon \triangle u + \overrightarrow{v} \cdot \overrightarrow{\nabla} u = 0$, defined in a rectangular domain $\Omega \equiv \{(x,y) \mid 0 \leq x \leq \pi a, 0 \leq y \leq \pi\}$, $a > 0$, with Dirichlet-type boundary conditions discontinuous at two of the corners of the domain $(0,0)$ and $(\pi a, 0)$: $u(x,0) = 1$, $u(x,\pi) = u(0,y) = u(\pi a, y) = 0$. This problem displays boundary and interior layers. We derive the exact solution of the problem by means of the method of separation of variables. The exact representation can be written in terms of a Fourier series which is transformed into a series of integrals in the complex plane from which we obtain complete asymptotic expansions. We approximate the solution by deriving asymptotic expansions from this series, not only in the singular limit $\varepsilon \to 0^+$ (with fixed distance to the discontinuity points $(0,0)$ and $(\pi a, 0)$), but also in the limit $r \to 0^+$ (with fixed ε), where r represents the distance to the points of discontinuity. Then, we approximate the solution on the whole domain, including the neighborhood of the points of discontinuity. It is shown that the first term of the expansion at $\varepsilon = 0$ contains a linear combination of error functions. This term characterizes the effect of the discontinuities on the $\varepsilon-$behaviour of the solution $u(x,y)$ in the boundary or the internal layers. On the other hand, near the points of discontinuity $(0,0)$ and $(\pi a, 0)$, the solution $u(x,y)$ is approximated by a linear function of the polar angle.

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Referencias

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[4] J.L. López y Ester Pérez Sinusía. *The role of the error function in a singularly perturbed convection-diffusion problem in a rectangle with corner singularities*. The Royal Society of Edinburgh Proceedings A (Mathematics), (2006).