# Optimal internal stabilization of the linear system of elasticity

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## Resumen

We address the nonlinear optimal design problem which consists in finding the best position and shape of the internal viscous damping set for the stabilization of the linear system of elasticity. Precisely, we consider the following nonlinear optimal design problem:

$$(P) \qquad \inf_{\omega\in\Omega_L} J\left(\mathcal{X}_\omega\right) = \frac{1}{2}\int_0^T \int_\Omega \left( \left|\boldsymbol{u}'\right|^2 + \boldsymbol{\sigma}(\boldsymbol{u}) : \boldsymbol{\varepsilon}(\boldsymbol{u}) \right) dx \tag{1}$$

where for a fixed $0 < L < 1$,

$$\Omega_L = \{\omega \subset \Omega : |\omega| = L\,|\Omega|\},$$

$|\omega|$ and $|\Omega|$ being the Lebesgue measure of $\omega$ and $\Omega$, respectively, and $\boldsymbol{u}$ is the solution of the elasticity system

$$\begin{cases} \boldsymbol{u}'' - \nabla_x \cdot \boldsymbol{\sigma} + a\,(x)\,\mathcal{X}_\omega\,(x)\,\boldsymbol{u}' = 0 & \text{in} \quad (0,T)\times\Omega, \\ \boldsymbol{u} = 0 & \text{on} \quad (0,T)\times\Gamma_0, \\ \boldsymbol{\sigma}\cdot\boldsymbol{n} = 0 & \text{on} \quad (0,T)\times\Gamma_1, \\ \boldsymbol{u}(0,\cdot) = \boldsymbol{u_0}, \quad \boldsymbol{u}'(0,\cdot) = \boldsymbol{u_1} & \text{in} \quad \Omega, \end{cases} \tag{2}$$

where $\mathcal{X}_\omega$ is the characteristic function of $\omega$, $\nabla_x\cdot$ is the divergence operator considered with respect to the spatial variable $x$, $\boldsymbol{n} = (n_1, \cdots, n_N)$ is the outward unit normal vector to $\Gamma_1$, $0 < T \le \infty$, and $a = a\,(x) \in L^\infty\,(\Omega)$ is a damping potential satisfying

$$a\,(x) \ge a_0 > 0 \quad \text{a. e. } x \in \omega.$$

Non-existence of classical designs are related to the over-damping phenomenon. Therefore, by means of Young measures, a relaxation of the original problem is obtained. Due to the vector character of the elasticity system, the relaxation is carried out through div-curl Young measures (see [3]) which let the analysis be direct and dimension independent.

The relaxed problem is solved numerically and a penalization technique to recover quasi-optimal classical designs from the relaxed ones is implemented in several numerical experiments. Finally, the influence of the Lamé coefficients is also studied numerically.

The results presented in this work generalize the ones obtained by the authors for the case of the scalar wave equation [1].

For full proofs of the results stated in this work see [2].

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### Referencias

[1] A. Münch, P. Pedregal and F. Periago, *Optimal design of the damping set for the stabilization of the wave equation,* J. Differential Equations **231** (2006) 331-358.

[2] A. Münch, P. Pedregal and F. Periago, *Optimal internal stabilization of the linear system of elasticity,* (2007) Preprint available at http://www.dmae.upct.es/ fperiago/.

[3] P. Pedregal, *Div-Curl Young measures and optimal design in any dimension*, Revista Mat. Complutense (in press).