AN OPTIMAL DESIGN PROBLEM IN WAVE PROPAGATION

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Resumen

In [4] a structural model was proposed for the design of finite structures made up of two given materials in the context of wave propagation. The model is an optimal design problem in which the distribution of two given materials is optimized so as to minimize a cost functional related to the vibration or propagation of waves along the medium. In practice, this model may be useful for the systematic design of wave filters, damping of waves, or wave guides. The authors develop a numerical method for the optimization of those structures based on topology optimization (see [3]), but there is no mathematical analysis of the model. In the numerical examples, they also observe the surprising fact that no microstructure appears between the two materials when one tries to design a filter or to minimize the vibration energy.

Our aim here is to analyze mathematically the model proposed in [4] in the one-dimensional situation for longitudinal vibration. More specifically, given two materials at our disposal with different Young's modulus and different density, the problem consists of finding the best distributions of the two initial materials in a rod in order to minimize the vibration energy in the structure under periodic loading. It is important to point out that we do not consider any volume constraint in this problem. This fact is in accordance with the physical nature of the problem, since waves propagate better through homogeneous materials than through mixtures of two materials. It is worthwhile to comment that the problem most related to our optimal design problem is the maximization of band gaps. Band gap materials prevent waves of frequencies belonging to a certain interval (the band gap interval) to propagate through the material. This is an optimal design problem in which an unit cell of a infinite periodic medium made up of two (given) materials is found in order to maximize the band gap interval. Rather than this, the aim of our problem is to design a finite structure (instead of a periodic infinite medium) in order to minimize the vibration energy of waves at particular values of the driving frequency.

Up to date, we have proved analytically the existence of classical solutions in certain cases, however, judging by the numerical simulations in [1], we can conjecture that the design problem admits optimal solutions in all of them. We have also dealt with minimizing the vibration energy along a tip-loaded cantilever beam for simple loading as well as multiple loading. Again, in view of the simulations ([2]), we also conjecture the existence of classical solutions for bending waves.

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Referencias

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[4] O. Sigmund and J.S. Jensen, *Systematic Design of Phononic Band-Gap Materials and Structures by Topology Optimization,* The Royal Society of London, Philosophical Transactions, Series A, Vol. 361, (2003), 65-96.