Matrix valued orthogonal polynomials satisfying differential equations

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Abstract

The theory of matrix valued orthogonal polynomials goes back to the fundamental works of M. G. Krein [3, 4]. If one is considering possible applications of these polynomials, it is natural to concentrate on those cases where some extra property holds. In [2] the problem of characterizing those positive definite matrix valued weights whose matrix valued orthogonal polynomials satisfy second order differential equations is raised. The scalar situation brings the very well known families of Hermite, Laguerre and Jacobi polynomials (see, for instance [1]). Nevertheless, the matrix case is entirely different. The noncommutative product and the existence of singular matrices make us think that we are very far away from a classification theorem.

In this communication, we will show recent advances in this subject, focusing on new phenomena that are not possible in the scalar case. For instance, we can obtain several linearly independent second order differential having a fixed family of orthogonal polynomials as eigenfunctions or we have found families of orthogonal polynomials satisfying odd order differential operators.

References

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