Fourier-Galerkin methods for boundary integral equations on axisymmetric bodies: theoretical and algorithmic aspects

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Resumen

In this work we present new results on the Fourier series boundary integral methods for axisymmetric domains. For these domains the usual integral equation on the boundary can be solved in an indirect way. Namely, the density is expanded in its Fourier series in terms of the angular variable. Then, each Fourier coefficient, which is itself a one-variable function, is in turn the solution of an integral equation, different for each coefficient, on the generatrix, i.e., the curve which generates the axisymmetric surface by rotation.

The advantages of this approach are clear. First, we have a reduction of the dimension of the problem. Instead of solving a 2D integral equation, arising itself from a 3D differential problem, we solve a 1D problem for each Fourier coefficient of the density. Secondly, there is no need to mesh the surface and we only have to work on the generatrix.

We restrict ourselves to the single layer equation for the Laplace operator. In spite of its simplicity, this problem retains the main features of the problem. Hence, it serves as a model problem to examine the difficulties we have to face and to test the numerical methods for solving the problem.

The functional properties of the line integral equations are analyzed. We prove that the ellipticity of the original problem is inherited by the line integral equations in adequate norms. Moreover, the corresponding operators turn out to be compact perturbations, in suitable spaces, of the single layer operator for the Laplace equation on the generatrix. This fact makes possible to extend the stability and convergence of some well-tested methods for integral equations on open arcs, such as collocation, spectral or quadrature methods, to the integral equations satisfied by the Fourier coefficients.

Two issues are also relevant for the applicability of the method. The first one is obtaining an estimate for the truncation error of the angular Fourier series. Having such an estimate makes possible to approximate the exact solution within a required tolerance avoiding the computation of an excessively large number of Fourier coefficients. The second one is to obtain a relation between the regularity of the unknown, in the original Sobolev spaces on the boundary, with the regularity of their Fourier coefficients on suitable weighted Sobolev spaces on the generatrix. Some results on these topics will be given.

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