Standing Waves of Some Coupled Nonlinear Schrödinger Equations

E. Colorado

Department of Mathematical Analysis, University of Granada, Granada 18071 (SPAIN)

colorado@ugr.es

A. Ambrosetti

International School for Advanced Studies (ISAS/SISSA), Trieste 34014 (ITALY)

ambr@sissa.it

Resumen

In spite of the interest that system of coupled NLS (Nonlinear Schrödinger) equations have in Nonlinear Optics, see e.g. [1], only few rigorous general results have been proved so far. Here, motivated by the recent paper [5], we deal with the system

$$\begin{cases} -\Delta u + \lambda_1 u = \mu_1 u^3 + \beta u v^2, & u \in W^{1,2}(\mathbb{R}^N) \\ -\Delta v + \lambda_2 v = \mu_2 v^3 + \beta u^2 v, & v \in W^{1,2}(\mathbb{R}^N) \end{cases} \tag{1}$$

where $\lambda_i, \mu_i > 0$, $i = 1, 2$, β is a real parameter and $x \in \mathbb{R}^N$, $N = 2, 3$.

We prove the existence of *bound* and *ground states* provided the coupling parameter $\beta < \Lambda$, respectively, $\beta > \Lambda'$, where $0 < \Lambda \le \Lambda' < \infty$. The main results are the following.

Theorem 1. If $\beta > \Lambda'$ then (1) has a (positive) radial ground state $\widetilde{\mathbf{u}}$.

Theorem 2. If $\beta < \Lambda$, then (1) has a radial bound state $\mathbf{u}^*$ such that $\mathbf{u}^* \ne \mathbf{u}_j$, $j = 1, 2$, where $\mathbf{u}_1 = (U_1, 0)$, $\mathbf{u}_2 = (0, U_2)$ and U_j is the positive radial solution to $-\Delta U_j + \lambda_j U_j = \mu_j U_j^3$. Furthermore, if $\beta \in (0, \Lambda)$, then $\mathbf{u}^* > 0$.

The main idea in the proof of Theorems 1, 2 is to show that the Morse index of $\mathbf{u}_1$ and $\mathbf{u}_2$ changes with β:

- for $\beta < \Lambda$ small their index is 1,
- while for $\beta > \Lambda'$ their index is greater or equal than 2.

This fact, jointly with an appropriate use of the *natural constraint method*, allow us to prove the existence of bound and ground states.

These results are announced in [2] and proved in [3].

Other results dealing with multi-bump solutions to linearly coupled systems of nonlinear Schrödinger equations will be commented at the end of the talk. See [4] for more details.

Sección en el CEDYA 2007: Ecuaciones en Derivadas Parciales (EDP).

Referencias

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[5] T-C. Lin & J. Wei, *Ground state of N coupled nonlinear Schrödinger equations in $\mathbb{R}^N$, $N \le 3$.* Comm. Math. Phys. **255** (2005), 629-653.