Standing Waves of Some Coupled Nonlinear Schrödinger Equations

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Resumen

In spite of the interest that system of coupled NLS (Nonlinear Schrödinger) equations have in Nonlinear Optics, see e.g. [1], only few rigorous general results have been proved so far. Here, motivated by the recent paper [5], we deal with the system

\begin{align}
-\Delta u + \lambda_1 u &= \mu_1 u^3 + \beta uv^2, \quad u \in W^{1,2}(\mathbb{R}^N) \\
-\Delta v + \lambda_2 v &= \mu_2 v^3 + \beta u^2 v, \quad v \in W^{1,2}(\mathbb{R}^N)
\end{align}

(1)

where \(\lambda_i, \mu_i > 0, i = 1, 2\), \(\beta\) is a real parameter and \(x \in \mathbb{R}^N, N = 2, 3\).

We prove the existence of bound and ground states provided the coupling parameter \(\beta < \Lambda\), respectively, \(\beta > \Lambda'\), where \(0 < \Lambda \leq \Lambda' < \infty\). The main results are the following.

**Theorem 1.** If \(\beta > \Lambda'\) then (1) has a (positive) radial ground state \(\tilde{u}\).

**Theorem 2.** If \(\beta < \Lambda\), then (1) has a radial bound state \(u^*\) such that \(u^* \neq u_j, j = 1, 2\), where \(u_1 = (U_1, 0), u_2 = (0, U_2)\) and \(U_j\) is the positive radial solution to \(-\Delta U_j + \lambda_j U_j = \mu_j U_j^3\). Furthermore, if \(\beta \in (0, \Lambda)\), then \(u^* > 0\).

The main idea in the proof of Theorems 1, 2 is to show that the Morse index of \(u_1\) and \(u_2\) changes with \(\beta\):

- for \(\beta < \Lambda\) small their index is 1,
- while for \(\beta > \Lambda'\) their index is greater or equal than 2.

This fact, jointly with an appropriate use of the natural constraint method, allow us to prove the existence of bound and ground states.

These results are announced in [2] and proved in [3].

Other results dealing with multi-bump solutions to linearly coupled systems of nonlinear Schrödinger equations will be commented at the end of the talk. See [4] for more details.

**Sección en el CEDYA 2007:** Ecuaciones en Derivadas Parciales (EDP).

**Referencias**


