Some qualitative results on magnetic vector fields

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Resumen

In this work I will focus on a class of vector fields which is particularly important in Physics and multidisciplinary applications, i.e. magnetic fields created by DC (direct current) flows. The mathematical description of these fields involve a smooth curve L in $\mathbb{R}^3$, parametrized by the embedding map $\tau : \mathbb{R} \to \mathbb{R}^3$, which represents the electric wire, and a constant J which stands for the current intensity. The magnetic field B evaluated at the point $r \in \mathbb{R}^3$ is obtained from Biot-Savart law, as was discovered in the XVIII century:

$$B(r) = \frac{\mu_0 J}{4\pi} \int_{-\infty}^{\infty} \frac{\dot{\tau}(t) \wedge (r - \tau(t))}{|r - \tau(t)|^3} \,, \tag{1}$$

where μ_0 denotes the magnetic permeability constant, the dot over τ stands for the derivative with respect to t and $\wedge$ and $|\cdot|$ represent the standard vector product and Euclidean norm in $\mathbb{R}^3$. If L is a closed curve then the integration sign must be substituted by $\oint$. Eq. (1) is the main formula of magnetostatics and yields the magnetic field created by the current distribution (L, J). According to the superposition principle the magnetic field created by n wires $(L_1, J_1), \ldots, (L_n, J_n)$ is given by the sum $B = \sum_{i=1}^{n} B_i$ of the individual magnetic fields B_i obtained from Biot-Savart law.

The results on this topic that I will report are the following:

1. Study of magnetic fields with Euclidean or radial symmetries: existence of first integrals and description of the phase portrait structure. I will also present the "non-swallowing" property for the motion of charged particles. These results are based on papers [3] and [4].

2. Examples of magnetic fields exhibiting the main features of Hamiltonian chaos: quasi-periodic orbits, KAM islands and homoclinic tangles. These results are based on paper [2].

3. Study of magnetic fields created by rectilinear configurations of wires: existence of first integrals and examples with open magnetic lines and chaos. I will also present some counterexamples to Stefanescu's conjecture which claims that certain kinds of magnetic fields possess a non-trivial polynomial first integral. These results are based on paper [1].

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Referencias

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[3] F. González-Gascón and D. Peralta-Salas, Motion of a charge in the magnetic field created by wires: impossibility of reaching the wires. Phys. Lett. A **333**, 72-78 (2004).

[4] F. González-Gascón and D. Peralta-Salas, Some properties of the magnetic fields generated by symmetric configurations of wires. Phys. D **206**, 109-120 (2005).