Connecting steady states of a diffusive energy balance climate model via controllability results

V. García, J.I. Díaz

Dpto. de Matemática Aplicada
Universidad Complutense de Madrid

victor.garcia.garrido@estumail.ucm.es and diaz.racefyn@insde.es

Resumen

In this communication we consider a simple Budyko-Sellers model of the type

$$(P)\begin{cases} y_t - (k(1-x^2)y_x)_x = R_a(y) - R_e(y,u) & x \in (-1,1), t > 0, \\ y(x,0) = y_0(x) & x \in (-1,1), \end{cases}$$

where $k > 0$, $R_a(y)$ is a bounded increasing function (the absorbed energy due to the co-albedo) and $R_e(y,u)$ is a strictly increasing function of the type $R_e(y,u) = u\,|y|^3\,y$ coming from the Stefan-Boltzman radiation law with an emissivity u which, varying in some positive interval, is taken here as a control variable (indicating the anthropogenerated actions on the rate of emissions on the greenhouse gases). It is well known (Hetzer (1990), Arcoya, Díaz and Tello (1998)) that the set of stationary solutions is very large (depending on the parameter u) and, for instance, it leads to a bifurcation diagram with a principal branch which is S-shaped containing at least one turning point to the right and another one to the left.

We consider the problem of transferring the system (through some sufficiently large time T) from a stationary state to another one in the same connected component. For instance, from an unstable state $(y_0(x), u_0)$ to a final stable one $(y_f(x), u_0)$ (near a turning point) of the principal branch. We are interested here in the case of possible localized controls of the form $u(t,x)\chi_{(l_1,l_2)}$, for some given latitude control interval $(l_1, l_2) \subset (-1, 1)$. We show that, at least, the transfer can be reached in a weak way (arriving as close as wished, in the L^2 norm, to the stable state). We also show that the transfer is exactly to the stable state by assuming some additional conditions. One of the main mathematical difficulties comes from the coupling term control/state $u\,|y|^3\,y$ (in contrast with other previous control formulations).

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