

Asymptotic Expansions of the Hurwitz-Lerch Zeta Function

CHELO FERREIRA

Dpto. de Matemática Aplicada, Universidad de Zaragoza

cferrei@unizar.es

JOSÉ L. LÓPEZ

Dpto. de Ingeniería Matemática e Informática, Univ. Pública de Navarra

jl.lopez@unizar.es

Resumen

The Hurwitz-Lerch Zeta function $\Phi(z, s, a)$ is considered for large and small values of $a \in \mathbf{C}$, and for large values of $z \in \mathbf{C}$, with $|\text{Arg}(a)| < \pi$, $z \notin [1, \infty)$ and $s \in \mathbf{C}$. This function is originally defined as a power series in z , convergent for $|z| < 1$, $s \in \mathbf{C}$ and $1 - a \notin \mathbf{N}$. An integral representation is obtained for $\Phi(z, s, a)$ which define the analytical continuation of the Hurwitz-Lerch Zeta function to the cut complex z -plane $\mathbf{C} \setminus [1, \infty)$. From this integral we derive three complete asymptotic expansions for either large or small a and large z . These expansions are accompanied by error bounds at any order of the approximation. Numerical experiments show that these bounds are very accurate for real values of the asymptotic variables.

Sección en el CEDYA 2007: otros temas

Referencias

- [1] T. M. Apostol, *On the Lerch zeta function*. Pacific J. Math., **1** (1951) 161-167.
- [2] D. Klusch , *On the Taylor expansion of the Lerch zeta-function* , J. Math. Anal. Appl., **170** (1992) 513-523.
- [3] D. Klusch , *Note sur la fonction $\mathcal{R}(w, x, s) = \sum_{k=0}^{\infty} e^{2k\pi ix} / (w + k)^s$* , Acta Math., **11** (1887) 19-24.
- [4] H. M. Srivastava , *Sums of certain series of the Riemann zeta function* , J. Math. Anal. Appl., **134** (1988) 129-140.