Convergence to equilibrium for a hyperbolic/elliptic system modelling the elastic-gravitational deformation of a layered Earth

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Resumen

In this communication we prove the stabilization, as t goes to infinity, of a model (which is an adaptation of the one possed by A. E. H. Love in 1911) for the study of the displacements due to internal sources of strain in layered linear elastic-gravitational continua. The coupled model of deformation and the variation of the gravity is the following system of partial differential equations:

$$\begin{cases} \rho \mathbf{u}_{tt} - \gamma \Delta \mathbf{u}_t - \Delta \mathbf{u} - \dfrac{1}{1-2\nu} \nabla (div \mathbf{u}) - \dfrac{\rho g}{\mu} \nabla (\mathbf{u} \cdot \mathbf{e}_z) + \dfrac{\rho g}{\mu} \mathbf{e}_z div \mathbf{u} = \dfrac{\rho}{\mu} \nabla \phi + \mathbf{f}_u, \\ -\Delta \phi = 4\pi \rho G div \mathbf{u} + f_\phi, \end{cases}$$

where $\mathbf{u}$ denotes the displacement, ν the Poisson's ratio, ρ the unperturbed density of the medium, g the externally imposed gravitational acceleration, μ is the rigidity and $\mathbf{e}_z$ is the unit vector pointing in the positive z-direction (down into the medium).

We consider a spatial domain of the type as shown in Figure 1. The elastic constants and the density of the nth layer are denoted by λ_n, μ_n and ρ_n. Each layer has thickness d_n. We construct a cylindrical coordinate system with the origin at the surface and with the z axis pointing down into the medium. The lower boundary of the nth layer is designated by z_n and the depth to the half space by z_p.

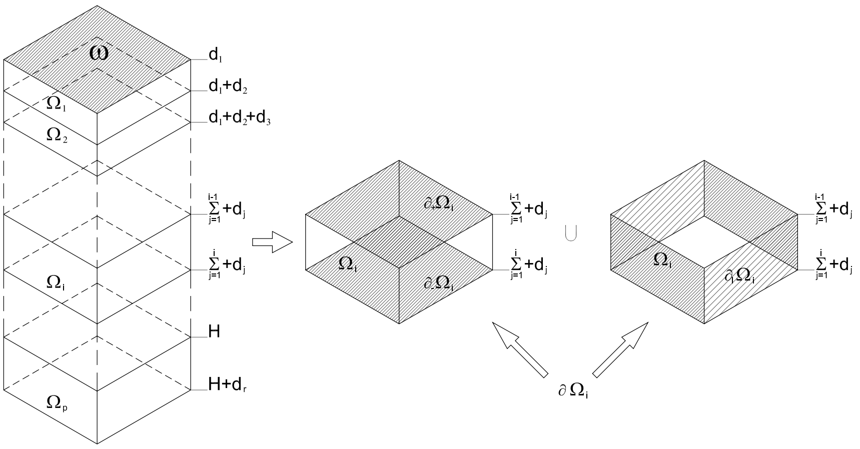

Figure 1. Layered Earth model. Illustration of the coordinate system and variation of the layer properties with depth.

The existence and uniqueness of weak solutions has been obtained recently in two different joint works by Arjona, Díaz, Fernández and Rundle (see also the DEA report by the first author at the UCM in July 2006). Here we prove that, under some additional conditions on the data, the difference of the respective solutions converges to zero, as t goes to infinity, in a suitable functional space. Our proof uses a reformulation of the hyperbolic/elliptic system in terms of a nonlocal hyperbolic system leading to a dynamical system to which we apply the La Salle invariance principle for a Lyapunov function involving the nonlocal terms.

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