An extinction delay mechanism for abstract semilinear equations

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Resumen

We consider the abstract semilinear problem

$$(ASP)\begin{cases} \frac{du}{dt} + Au + F(u) = G(t,u_t) & t \in (0,T), \\ u(s) = u_0(s) & s \in (-\tau, 0)\,, \end{cases}$$

where $T > 0$, $A : D(A) \to X$ is a linear m-accretive operator on a Banach space X, $F : X \to X$ is a C^1 function and $G : [0,T) \times C([-\tau,0] : X) \to X$ is a suitable delayed action, where $u_t(s,\cdot) := u(t+s,\cdot)$ for $s \in [-\tau, 0]$).

We make use of a nonlinear variation of constants formula (the Alekseev formula) to show that given $\tau \in (0, T/2]$, A and F, for any initial datum $u_0(s)$ (in fact we only need to know the value at time $s = 0$, $u_0(0)$) there exists a delayed action $G(t,u_t)$ such that the solution of (ASP) becomes extinct after the time 2τ, i.e. $u(t) = 0$ in X for any $t \geq 2\tau$.

We also show that in the linear case ($F(u) \equiv 0$) the conclusion holds when the delated action is taken (independently of the initial datum $u_0(s)$) of the form $G(t,u_t) = -b(t)u_t(\tau,\cdot)$ for a suitable real function $b(t)$ which becomes extinct after 2τ (typical of switched controls), $b(t)$ being inactive (i.e. zero) on $[0,\tau]$ and satisfying that $1 = \int_\tau^{2\tau} b(s)ds$.

We recall that, in contrast with most of the previous "finite extinction results" for parabolic reaction-diffusion equations (where usually it is required that F be a non Lipschitz nonlinear term and $G(t,u_t) \equiv 0$), the term $F(u)$ is assumed here to be a smooth function.

The results generalize a previous work by the authors (to appear in DYNAMICS OF CONTINUOUS, DISCRETE AND IMPULSIVE SYSTEMS (Series A)) dealing with linear scalar parabolic equations. By taking suitable choices of the Banach space X, of the operator A and function F, we apply here the abstract result to some other models such as, for instance, the case of general linear abstract equations and some nonlinear damped hyperbolic equations, to mention only two of the many possible applications.

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