A stabilized difference scheme for deformable porous media and its numerical resolution on block-structured grids by multigrid methods

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Resumen

The classical quasi–static Biot model [1] for soil consolidation, describes mathematically the time dependent interaction between the deformation of an elastic porous material and the fluid flow inside of it. This model can be formulated as a system of partial differential equations for the unknowns displacement and pressure. By $\mathbf{u} = (u, v)$ we denote the displacement vector and by p the pore pressure of the fluid. Here, we consider the case of a homogeneous, isotropic and incompressible medium Ω, so the governing equations are given by

$$-\mu\tilde{\Delta}\mathbf{u} - (\lambda + \mu)\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \mathrm{grad}\,p = \mathbf{g}(\mathbf{x}, t), \tag{1}$$

$$\frac{\partial}{\partial t}(\mathrm{div}\,\mathbf{u}) - \frac{\kappa}{\eta}\Delta p = f(\mathbf{x}, t), \quad \mathbf{x} \in \Omega, \quad 0 < t \leq T, \tag{2}$$

where λ and μ are the Lamé coefficients, κ is the permeability of the porous medium, η the viscosity of the fluid and $\tilde{\Delta}$ represents the vectorial Laplace operator. The quantity $\mathrm{div}\,\mathbf{u}\,(\mathbf{x}, t)$ is the dilatation, i.e. the volume increase rate of the system, which can be considered as a measure of the change in porosity of the soil. The source terms $\mathbf{g}(\mathbf{x}, t)$ and $f(\mathbf{x}, t)$ are used to represent a density of applied body forces and a forced fluid extraction or injection process respectively.

When a load is applied on an elastic and saturated porous medium, the pressure suddenly increases and a sharp boundary layer can appear in the early stages of the time-dependent process. In the case of an unstable discretization, unphysical oscillations appear. A stabilized finite difference scheme on collocated grids based on the perturbation of the flow equation was proposed in [2], which provides us solutions without oscillations independently of the chosen discretization parameters.

When discretizing the incompressible poroelasticity equations with standard second order central differences and an artificial pressure term, the development of multigrid smoothing methods is not straightforward. Smoothing factors of standard collective point-wise relaxations are not satisfactory. A possibility to overcome this problem is to extend the idea of box relaxation to the non-staggered case. By other hand, if grid applications are to be implemented on parallel computers, grid partitioning is a natural approach. In this approach, the original grid is split into P subdomains such that P available processors can jointly solve the discrete problem. Grid partitioning is the natural parallelization for multigrid. We present here an efficient multigrid solver for the poroelasticity problem with block-structured grids in a grid partitioning environment.

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Referencias

[1] M. Biot, *General theory of three dimensional consolidatio*, J. Appl. Phys. **12** (1941) 155-169.

[2] F. J. Gaspar, F. J. Lisbona, C. W. Oosterlee, and P.Ñ. Vabishchevich, An efficient multigrid solver for a reformulated version of the poroelasticity system. *C*omput. Methods Appl. Mech. Engrg. 2007; **196**:1447–1457.