Finite difference approximation for secondary consolidation problems and its numerical resolution by multigrid

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Resumen

Soil consolidation theory addresses the time dependent coupling between the deformation of a porous matrix and the fluid flow inside. The porous matrix is supposed to be saturated by an incompressible fluid phase and the flow governed by Darcy's law. The state of the continuous medium is characterized by the knowledge of the displacements and the fluid pressure at each point of the domain. The consolidation process under one dimensional conditions was first investigated by Therzaghi [4] and a phenomenological model for a rather general situation was proposed and analyzed by Biot [1] in three dimensions. These authors assumed an elastic response of the soil skeleton to the loads. Under the previous assumption a change in stress will generate a deformation and an excess of pore pressure. The dissipation of the pressure will result into a final deformed state. This is not the situation in some cases where the soil deformation continues even though all excess pore pressures have been dissipated. The presence of this process, the so called secondary consolidation, is typical in the consolidation of clay soils. A mathematical model has been formulated by Murad and Cushman in [2] and reported by Showalter in [3].

This Biot's type model is given by a system of partial differential equations for the unknown displacement and pressure. We denote by $\mathbf{u}$ the displacement vector and by p the pore pressure of the fluid. The governing equations for a homogeneous, isotropic and incompressible medium Ω read

$$\begin{aligned} -\lambda^* \nabla \frac{\partial}{\partial t}(\nabla\cdot\mathbf{u}) - \mu\tilde{\Delta}\mathbf{u} - (\lambda+\mu)\nabla\,(\nabla\cdot\mathbf{u}) + \alpha\,\nabla p = \mathbf{0}, \\ \alpha\frac{\partial}{\partial t}(\nabla\cdot\mathbf{u}) - \kappa\Delta p = f(\mathbf{x},t), \end{aligned} \tag{1}$$

where λ^* is a secondary consolidation parameter, $\tilde{\Delta}$ is the vectorial Laplace operator, λ and μ are the Lamé coefficients, κ is the hydraulic conductivity, α is the Biot-Willis coefficient and f represents a forced extraction or injection process.

For the numerical approximation of these equations we use finite difference schemes on staggered grids such that the main properties of the differential operators are preserved in the discrete level. Also, a weighted two–level discretization on a staggered mesh has been adopted for time stepping which is coherent with the lack of initial condition for the pressure. A priori estimates in discrete energy norm are obtained and convergence results are given.

Finally, we also introduce an efficient multigrid method for this problem. In particular, we present a pointwise smoothing method based on distributive iteration. In distributive smoothing the original system of equations is transformed by pre-conditioning in order to achieve favourable properties, such as a decoupling of the equations and/or possibilities for pointwise smoothing. Numerical experiments confirm the theoretical results and efficiency of the proposed solver.

Sección en el CEDYA 2007: AN

Referencias

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