Null controllability results for parabolic equations in unbounded domains

M. González-Burgos

Dpto. Ecuaciones Diferenciales y Análisis Numérico, Univ. de Sevilla

manoloburgos@us.es

L. de Teresa

Instituto de Matemáticas, Universidad Nacional Autónoma de México

deteresa@matem.unam.mx

Resumen

In this talk we present some results concerning the null controllability of the system

$$\begin{cases} y_t - \Delta y + B \cdot \nabla y + ay = v1_\omega \text{ in } Q = \Omega \times (0,T), \\ y = 0 \text{ on } \Sigma = \partial\Omega \times (0,T), \quad y(x,0) = y_0(x) \text{ in } \Omega, \end{cases} \tag{1}$$

posed in an **unbounded** domain $\Omega \subset \mathbb{R}^N$. In (1), 1_ω denotes the characteristic function of the subset $\omega \subset \Omega$; $B \in L^\infty(Q)^N$, $a \in L^\infty(Q)$ and $y^0 \in L^2(\Omega)$ are given, $y = y(x,t)$ is the state and $v = v(x,t)$ is the control function (which acts on the system through the subset ω). It is by now well known that (1) is, in general, not null controllable at time $T > 0$ (see [4, 5, 6]).

We first present a global Carleman inequality for the adjoint problem (and then, a positive null controllability result for system (1)) under appropriate assumptions on Ω and ω. We also give some examples of unbounded domains (Ω, ω) that satisfy these sufficient conditions. As a consequence, we will be able to prove the results of [1], [2] and [6] about the null controllability in $L^2(\Omega)$ of system (1) when $a \in L^\infty(Q)$ and $B \in L^\infty(Q)^N$.

Secondly, we will analyze the controllability properties of system

$$\begin{cases} y_t - \Delta y + f(y, \nabla y) = v1_\omega \text{ in } Q, \\ y = 0 \text{ on } \Sigma, \quad y(x,0) = y_0(x) \text{ in } \Omega, \end{cases} \tag{2}$$

when Ω is an unbounded domain, $\Omega \backslash \omega$ is a bounded set and the nonlinearity $f(y, \nabla y)$ grows slower than $|y| \log^{3/2}(1 + |y| + |\nabla y|) + |\nabla y| \log^{1/2}(1 + |y| + |\nabla y|)$ at infinity (generally in this case in the absence of control, blow up occurs). We will obtain the result on null controllability for system (2) stated in [3] for **bounded domains** Ω and, in particular, we will generalize the result on null controllability stated in [1] for globally Lipschitz-continuous functions f.

Sección en el CEDYA 2007: CO

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