Reproductive and time periodic solutions for incompressible fluids

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Resumen

We study some problems related with time periodic solutions for models of incompressible fluids.

We recalling the main ideas to prove the existence of reproductive weak solutions (i.e. weak solutions defined in the time interval $(0, T)$ taking the same initial and final values in time) for the Navier-Stokes equations and some variants where these ideas are applicable, such as Boussinesq, micropolar and magneto-micropolar models. This proof relies on the obtention of time periodic Galerkin approximations via Leray-Schauder point fixed argument.

Moreover, in the case of $2D$ domains, using the uniqueness of weak solutions, the regularizing property of the system and the existence of global regular solutions when data are regular, one has that the periodic in time weak solutions defined as extension of reproductive solutions to the whole time interval $(0, +\infty)$ will be regular solutions. An extension of these results to the $3D$ case is possible imposing small enough external force, using the so called "weak/strong uniqueness" and the global strong solutions for small enough data.

Also, we study some coupled models for velocity and pressure dynamic variables with another variable where the maximum principle holds, such as the generalized Boussinesq model (with temperature-dependent viscosity) and a nematic liquid crystal model with a Ginzburg-Landau penalization. In these cases one has, thanks to an adequate reformulation of the problem by truncation, existence of reproductive weak solutions as limit of time periodic Galerkin approximations. It is important to remark that Galerkin approximations do not verify the maximum principle but their limit does.

Finally, we will see that, for these models related with the maximum principle, the argument to prove regularity of reproductive solutions in the Navier-Stokes framework are not valid in general. The particular case of generalized Boussinesq model with Neumann boundary condition for the temperature can be solved with other arguments, but the case of nematic liquid crystal model remains as an open problem.

Sección en el CEDYA 2007: EDP

Referencias

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