The numerical analysis of higher-order nonlinear FE method for advection dominated problems

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Resumen

It is well known that numerical simulations of convection-dominated problems present numerical difficulties related to the lack of stability: because convection dominates diffusion, classical Galerkin finite element (FE) methods generate unstable approximations, which usually exhibit spurious oscillations. The SUPG (streamline upwind Petrov-Galekin) method, proposed by Brooks and Hughes [1], was the first variationally consistent, stable and accurate finite element model for advection dominated problems. For regular solutions this method presents quasi-optimal rates of convergence for the streamline derivative and was first analyzed by Johnson et al. [4]. Nevertheless, for nonregular solutions, localized oscillations are still observed in the neighborhood of steep gradients meaning that the streamline is not always the appropriate upwind direction. To overcome this lack of monotonicity many discontinuity capturing terms were designed to enhance stability either in a linear [5] or nonlinear way [2, 3]. Quite promising results were obtained using the nonlinear CAU (Consistent Approximate Upwind) finite element method proposed in [2]. The systematic way of updating the upwind direction in the CAU method results in adding to the SUPG formulation a nonlinear discontinuity-capturing term in a consistent way, engendering an additional stability in the direction of the approximate gradient. The theory has been refined over the years in several directions.

An important improved version of the CAU method, even for higher-order elements, has been discussed in the recent paper [3]. In this work a stability analysis was shown based on a linearized iterative scheme, which uses the solution of the SUPG method as an initial guess in order to solve the CAU (nonlinear) method. However, this analysis has some open ends [6] concerning the solvability of the iteration scheme. Therefore, our main goal is to improve the developed analysis. In particular, we address the problem associated to the convergence of the linearized iterative scheme as well as the existence of the solution for the nonlinear approximate method. The theoretical results will be supported by some numerical experiments.

Palabras clave: advection-diffusion equations, nonlinear finite element method, numerical analysis.

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