Reduction methods for quasilinear differential-algebraic equations

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Resumen

Quasilinear autonomous differential-algebraic equations (DAEs) are implicit ODEs of the form

$$A(x)\dot{x} = f(x), \tag{1}$$

where $A \in C^\infty(\Omega_0, \mathbb{R}^{n\times n})$ is a rank-deficient matrix-valued mapping, $f \in C^\infty(\Omega_0, \mathbb{R}^n)$, and Ω_0 is an open set in $\mathbb{R}^n$. Geometric methods for DAEs aim at an iterative reduction of the problem to an explicit ODE on a lower-dimensional submanifold of Ω_0. This approach can be traced back to the seminal work of Dirac on generalized Hamiltonian dynamics [1, 2], and was later developed by Rheinboldt [7], Reich [5, 6], and Rabier and Rheinboldt [3, 4].

Reduction methods are usually based on certain algebraic (typically constant-rank) conditions holding at every reduction step. When these conditions are met the DAE is called *regular*. We will discuss in this talk several recent results concerning the use of reduction techniques in the local classification problem for DAEs under contact equivalence, not only for regular systems but also for *singular* ones, where the above-mentioned conditions fail. Related dynamical aspects will also be addressed.

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Referencias

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