Fifth order accurate numerical approximation of Hamilton-Jacobi equations

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Resumen

We present a class of Weighted Power-ENO (Essentially Non Oscillatory) schemes to approximate the viscosity solutions ([1, 2, 6]) of Hamilton-Jacobi equations. Hamilton-Jacobi equations appear in many applications, for example, geometrical optics, optimal control, differential games, material sciences and calculus of variations. Therefore, it is essential to develop efficient, high order accurate numerical methods to solve these equations.

The Weighted Power-ENO schemes ([9]) were originally developed for hyperbolic conservation laws. The essential idea of the Power-ENO reconstruction is to apply a class of extended limiters ([8, 9]) to second order differences in the classical third order ENO ([3, 7]) reconstruction, so that the reconstruction is able to retain more information of the fine scales of the solution and improve resolution near discontinuities of the solution. A weighting strategy based on appropriate smoothness indicators ([5]) is then used to improve the reconstruction to be fifth order accurate; this is the so-called Weighted Power-ENO reconstruction. Here we adapt such Weighted Power-ENO reconstruction to Hamilton-Jacobi equations,([10]).

Our high order numerical schemes for Hamilton-Jacobi equations consist of three ingredients: a monotone numerical Hamiltonian, the fifth order Weighted PowerENO reconstruction and a fourth order strong stability preserving Runge-Kutta time stepping procedure, [11].

We give extensive numerical examples to demonstrate the accuracy, robustness and the resolution capability of the new scheme including a 2-D non-convex Riemann problem, a problem related to optimal control, and an application of high order accurate *level set* reinitialization using noisy distance functions as initial data.

In comparison to the standard fifth order WENO scheme ([4, 5]), the resulting scheme enjoy similar overhead and have much better capability of resolving viscosity solutions near kinks where the solution has discontinuous gradients as illustrated in the numerical examples.

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