Phase portraits of separable Hamiltonian systems

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Resumen

We study some generalizations of potential Hamiltonian systems ($H(x,y) = y^2 + F(x)$) with one degree of freedom. In particular, we are interested in Hamiltonian systems with Hamiltonian functions of type $H(x,y) = F(x) + G(y)$ or $H(x,y) = y^2\, G(x) + F(x)$, both arising in applied mechanical problems. We present an algorithm to plot the phase portrait of any Hamiltonian system of type $H(x,y) = F(x) + G(y)$, where F and G are arbitrary polynomials. We are able to give the full description in the Poincaré disk according to the graphs of F and G, extending the well-known method for the "finite"phase portrait potential systems.

In particular, the algorithm allows to establish the boundedness of the basins of attraction of the centers, which gives some information about the period function of those centers. In the frequent case of coexistence of centers, it is an interesting problem to know the "simultaneous"period functions. In the family of systems that we study, and for which we have a control of the centers' locations, we also study (using the techniques introduced in [1]) the relationship between the period functions of different centers. This can give, for the mechanical systems associated, an idea of the duration of the oscillations according to the initial positions.

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Referencias

[1] E. Freire, A. Gasull, A. Guillamon *Phase portrait of First derivative of the period function with applications*, Journal of Differential Equations **204** (2004), 139–162.