Qualitative features of Hamiltonian systems through averaging and reduction

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Resumen

Let us consider a small parameter which is a measure of the perturbation of an integrable system where all the solutions are periodic. Then let us normalize (or average) the perturbation term by term in the small parameter. After a finite number of terms have been normalized the higher order perturbations are truncated thus obtaining an approximation of the full system. This approximation is well defined on the lower dimensional reduced space. Being lower dimensional, sometimes just two-dimensional, the system on the reduced space is easier to understand, see for example [2]. By studying the reduced flow it is possible to obtain information on the existence, stability, and bifurcation of periodic solutions of the departure problem. It is even possible to get approximate invariant tori and other higher dimensional invariant structures. However, not all the features of the reduced system accurately portray the original full system. It typically does not see the breakdown of invariant tori, ergodic regions, solenoids, etc.

We shall illustrate the case of regular and singular reductions pointing out the differencies and peculiarities of each of them.

The goal of this presentation is to state results which have hypotheses on the reduced system and have conclusions about the full system and apply them to an example of a restricted three-body system. We shall perform two reductions, first a regular one and then a singular one. We will discuss then existence of families of periodic orbits, families of KAM tori and bifurcations of them extracted from the two reduction processes.

The theory has been developed in [1].

Sección en el CEDYA 2007: EDO

Referencias

[1] H.S. Dumas, K. Meyer, J.F. Palacián and P. Yanguas, *Averaging in Celestial Mechanics: Regular Case*, submitted, 2007.

[2] K. R. Meyer and G. R. Hall, *Introduction to Hamiltonian Dynamical Systems and the N-Body Problem*, Appl. Math. Sci. 90, Springer-Verlag, New York, 1992.