Error estimates of optimal order in a fractional-step scheme for the $3D$ Navier-Stokes equations

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Resumen

We present some improvements on the error estimates obtained by J.Blasco and R.Codina [**?**, **?**] for a viscosity-splitting in time scheme, with finite element approximation, applied to the Navier-Stokes equations. The key is to obtain new error estimates for the discrete in time derivative of velocity, which let us to reach, in particular, error of order one (in time and space) for the pressure approximation.

The unsteady, incompressible Navier-Stokes equations in a bounded domain $\Omega \subset I\!R^3$ is:

$$(P) \qquad \begin{cases} \mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} - \nu\,\Delta\mathbf{u} + \nabla p = \mathbf{f}, \quad \nabla\cdot\mathbf{u} = 0 \;\text{ in } \Omega\times(0,T), \\ \mathbf{u} = 0 \;\text{ on } \partial\Omega\times(0,T), \quad \mathbf{u}_{|t=0} = \mathbf{u}_0 \;\text{ in } \Omega, \end{cases}$$

where $\mathbf{u}(\mathbf{x},t)$ is the velocity of the fluid at position $\mathbf{x}\in\Omega$ and time $t\in(0,T)$, $p(\mathbf{x},t)$ the pressure, $\nu > 0$ the viscosity (which is assumed constant) and $\mathbf{f}$ the external force.

We will study a viscosity-splitting scheme, introduced and studied by J.Blasco and R.Codina [**?**, **?**, **?**]. It is a two-step scheme, where the main numerical difficulties of (P) (namely, the treatment of nonlinear term $(\mathbf{u}\cdot\nabla)\mathbf{u}$ and the relation between incompressibility $\nabla\cdot\mathbf{u} = 0$ and pressure), are split into two different steps. On the other hand, the diffusive term is considered in both steps, which allows to enforce the original boundary conditions of the problem in the two steps of the scheme, contrary with the well known projection schemes [**?**, **?**].

We consider a partition $\{t_n = n\,k\}$ with $k = T/N$ of the time interval $[0,T]$ and a regular triangularization of the domain Ω of mesh size h jointly with a "inf-sup"stable finite element scheme of order at least one.

Starting of the error estimates obtained in [**?**, **?**] and assuming additional regularity hypotheses on the exact solution, the objectives of this work are:

1. To improve the order of error estimate in pressure in norm $l^2(L^2)$, from $O(\sqrt{k}+h/\sqrt{k})$ to $O(k+h)$,

2. To improve the norm of error estimates in velocity and pressure, concretely from $l^\infty(\mathbf{L}^2)$ to $l^\infty(\mathbf{H}^1)$ in velocity and from $l^2(L^2)$ to $l^\infty(L^2)$ in pressure,

3. To improve the order in space of error estimates in velocity in norm $L^2(\mathbf{L}^2)$, from $O(k+h)$ to $O(k+h^2)$,

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