On a free boundary morpho-dynamic problem in landscape evolution

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Resumen

This contribution is devoted to the derivation and mathematical analysis of a morpho-dynamic problem of landscape evolution which has been recently addressed in geo-mathematics (see [1]). An overland flow over an erodible substrate is considered and the key process affecting hillslope morphology (soil erosion by water which causes detachment, transport and deposition of sediments) is modeled. This approach leads to a singular free boundary problem for a second order quasilinear parabolic equation derived from fundamental conservation laws. We start with the *strong formulation* suggested in ([1]) of the Cauchy problem for an initial thickness perturbation $h_0(x)$, say a bounded and non negative function $h_0(x)$ with a compact and connected support $(\xi_-(0), \xi_+(0))$ and such that $h_0^m(x)$ has a prescribed total mass M. We shall be especially interested in the question of *global solvability* of the following problem.

Given $m > 1$, $T \in I\!R^+$ (eventually $T = +\infty$) and $M \in I\!R^+$, find two continuous curves $\xi_-, \xi_+ : [0, +\infty) \to I\!R$ and a function $h : \mathcal{P}_T \to [0, +\infty)$ where $\Omega_0 = (\xi_-(0), \xi_+(0))$, $\Omega_t = (\xi_-(t), \xi_+(t)) \times \{t\}$, $\mathcal{P}_T = \cup_{t>0}\Omega_t$, such that

$$\begin{cases} h_t = (h^m)_{xx} + h^m, & \text{in } \mathcal{D}'(\mathcal{P}_T), \\ h(x,0) = h_0(x) & \text{a.e. } x \in \Omega_0, \\ h(x,t) > 0, & \text{a.e. } (x,t) \in \mathcal{P}_T, \\ h(x,t) \equiv 0, & \text{a.e. } (x,t) \notin \mathcal{P}_T, \\ h(\xi_-(t),t) = h(\xi_+(t),t) = 0, & \text{for any } t \in (0,+\infty), \\ \xi_-(0) = \xi_-^0,\ \xi_+(0) = \xi_+^0 \text{ and } \xi_-(t) < \xi_+(t) & \text{for any } t \geq 0, \\ \displaystyle\int_{\xi_-(t)}^{\xi_+(t)} h^m(x,t)dx = M & \text{for any } t \in (0,+\infty). \end{cases}$$

This is a free boundary problem for a nonlinear heat equation with source while the last equation is a conservation law written in term of an integral (non local) constraint which fix the free (moving) boundaries $\xi_-(t)$ and $\xi_+(t)$ separating the (connected) region where $h(x,t) > 0$ from the region where $h(x,t) = 0$. Notice that the mass conservation constraint prevents the blow-up phenomenon. The solution to this problem should be understood in a suitable weak sense that we shall make precise when dealing with the weak global (or complementary) formulation associated to the above strong formulation. Our aim is to show that the strong formulation of [1] can be written in terms of a non homogeneous condition for the flux at the free boundaries. This allows to get an unconstrained global weak formulation for which we propose a convergent iterative algorithm leading to the existence of solutions. Some numerical simulation results will be also presented.
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Referencias

[1] Fowler, A.C., Natalia Kopteva and Charles Oakley 2005 *The formation of river channels.* SIAM J. Appl. Math., in press.