A domain decomposition method derived from the Primal Hybrid Formulations for 2nd order elliptic problems

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Resumen

We introduce a new domain decomposition method obtained when the classical iterative method of Uzawa is applied to the primal hybrid formulation for second order elliptic problems. In this formulation the Lagrange multipliers that inforce the continuity of the approximations across interfaces are expressed via the duality $H^{-1/2} - H^{1/2}$, see for instance the works of Raviart-Thomas [5], Roberts-Thomas [6]. Usually, for numerical discretizations, this duality is worked out by means of some projection operator onto the L^2 space on the interfaces, see for instance work of Ben Belgacem [2]. In our approach we use Riesz representation and replace the duality with the $H^{1/2}$ scalar product that is explicitly computed. As a consequence, we have a formulation in terms of a saddle point problem suitable for iterative techniques, see for instance, the recent survey by Bacuta [1].

The coupling of the different subdomains is performed through the Lagrange multipliers while the coercive form in the formulation does not relate these different subdomains. Therefore, we have observed that the application of Uzawa algorithm yields a domain decomposition method, geometrically convergent with a mesh independent ratio. This property is shared with other well known methods like the Dirichlet-Neumann method proposed by Marini-Quarteroni [4] and the one by Lube-Müller-Otto [3].

The computation of the $H^{1/2}$ scalar product for the discrete basis functions on the interface is performed once as long as the mesh does not change on it. Comparing this computational work with the accuracy benefit that we obtain we believe it is worthwhile for applications were the interfaces are not complicated. Alternative iteration techniques, like conjugate gradient method and the inexact Uzawa algorithm will be considered in future works. While the augmented Lagrangian method couples different subdomains and therefore does not seem of interest, the study of the conjugate gradient method and the inexact Uzawa algorithm are promising, see Bacuta [1]. Different applications will also be studied in the future.

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