The Restricted 3-Body Problem on S^1: regularization and a particular solution.

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Resumen

One of the most famous problems in Mathematics is *The 3-Body Problem*. It consists in discribing the dynamics of three punctual masses in an euclidean space, interacting among themselves through no other forces than their mutual gravitational attraction according to Newton's law. Due to the difficulty of founding the explicit solutions to this problems, there are simpler formulations. In 1772, Euler proposed the most studied of these, known as *The Restricted 3-Body Problem*, which is a limit case of *The 3-Body Problem*. It considers two masses depicting circular paths about their center of mass with constant angular velocity on a plane, and the third is a negligible mass such that the formers don´t realize of the existence of the latter. The objective is to determine the dynamics of the negligible mass (see Boccaletti et. al. [1]). Nowadays, there no more than partial results in these problems, so here we present a variant of *The Restricted 3-Body Problem*.

First, we begin defining *The 2-Body Problem on* S^1 and we regularize it through a coordinate transformation that includes a change on time variable, following Érdi [2]. Then we show the global dynamics for all time.

Second, we state *The Restricted 3-Body Problem on* S^1 and clasify it in four cases: *the elliptic*, *the parabolic* and *the hyperbolic* cases and the case *with two fixed centers*, according to solutions of *The 2-Body Problem on* S^1. Afterwards, we regularize all the singularities due to binary collisions through one transformation of coordinates and a time-rescaling. The hamiltonian structure of the problem is preserved and we get a hamiltonian which depicts the dynamics of the negligible mass for all time while there is no total collision.

Finally, we outline the global dynamics of *The Restricted 3-Body Problem on* S^1 *with two fixed centers* and exhibit a particular solution that is free from binary collisions and happens only in the *hyperbolic* case.

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Referencias

[1] D.Boccaletti, G.Puccaco, *Theory of Orbits*, Vol.1, Springer-Verlag Berlín, Alemania, 2001.

[2] Bálint Érdi, *Global Regularization of the Restricted Problem of Three Bodies*, Celestial Mechanics and Dynamical Astronomy **90**: 35-42, 2004.