Regularization and relaxation tools for interface coupling

F. Coquel, E. Godlewski, N. Seguin

Laboratoire Jacques-Louis Lions, Univ. Pierre et Marie Curie & CNRS, Paris

coquel@ann.jussieu.fr, godlewski@ann.jussieu.fr, seguin@ann.jussieu.fr

Resumen

We have been considering, in a series of papers, the coupling of systems, both from a theoretical and from a numerical point of views. The actual problem arises from the coupling of codes in the industrial context of thermohydraulics in nuclear reactors. These codes model liquid-vapor flows and the systems under consideration are systems of conservation laws of hyperbolic nature. Typically, as model problems, we have been considering the coupling of homogeneous models of two-phase flows (HEM and HRM models in [1]).

The coupling problem might be interpreted as solving conservation laws with discontinuous coefficients, in which case the flux is assumed to be continuous at the interface. However, we have followed an approach where the coupling is a priori non conservative, which we have named *state coupling* since (in general) it ensures the continuity or *transmission* of the state variables, as opposed to *flux coupling*.

The precise coupling conditions, introduced in [5], impose that two boundary value problems be well-posed. Since boundary value problems for hyperbolic systems are a difficult subject, these coupling conditions cannot always be explicited, and moreover may lead to ill-posed problems. We have been able to justify the approach from a theoretical point of view, using different tools and in various contexts.

The special structure of Lagrangian systems [2] has enabled us to express the coupling condition when a special set of transmitted variables is chosen, for rather general fluid systems.

Then, Dafermos regularization was introduced in [4] in the scalar case. This regularization allows the approximation of coupled Riemann problems by smooth profiles and enlights the possible discontinuous behavior of the limit solutions at the interface.

In another direction, we have used relaxation systems in a global numerical procedure. By introducing a larger but simpler system on which a state coupling procedure is applied, it ensures a conservative numerical coupling of two Euler systems which avoids resonance. This very powerful tool can be used for rather general fluid systems. In the scalar case, it can be analyzed and justified and we can link the results to those of [3].

Sección en el CEDYA 2007: EDP, AN

Referencias

[1] A. Ambroso, C. Chalons, F. Coquel, E. Godlewski, F. Lagoutière, P.-A. Raviart, N. Seguin. *The coupling of homogeneous models for two-phase flows.* Int. Journal for Finite Volume, **4**(1) (2007).

[2] A. Ambroso, C. Chalons, F. Coquel, E. Godlewski, F. Lagoutière, P.-A. Raviart, N. Seguin. *Coupling of general Lagrangian systems* (submitted) (2006).

[3] F. Bachmann, J. Vovelle. *Existence and uniqueness of entropy solution of scalar conservation laws with a flux function involving discontinuous coefficients.* Comm. Partial Differential Equations, **31** no. 1-3 (2006), 371–395.

[4] B. Boutin, F. Coquel, E. Godlewski. *Dafermos regularization for interface coupling of conservation laws.* In Eleventh International Conference on Hyperbolic Problems, Theory, Numerics, Applications. *Hyp2006 proceedings*, Springer (2007).

[5] E. Godlewski, K.-C. Le Thanh, P.-A. Raviart. *The numerical interface coupling of nonlinear hyperbolic systems of conservation laws: II. The case of systems.* ESAIM:M2AN, **39** (2005), 49–692.