Introduction to the theory of non-autonomous and stochastic/random dynamical systems

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Resumen

The theory of autonomous dynamical systems and their attractors is well established. Essentially, the mechanism driving the dynamics does not change in time, and the dynamics can be described by a group or semi-group of mappings taking the state space into itself and attractors are compact sets which are invariant, i.e. mapped onto themselves, and attract all nearby trajectories. A particular characteristic of autonomous systems is that only the elapsed time since starting is important, not the actual and starting times themselves.

In non-autonomous dynamical systems and random dynamical systems the driving mechanism itself changes in time, which must also be built into the system description. The most obvious way to do this is to describe the dynamics by a two-parameter group of mappings of the state space into itself, with the parameters being the actual time and the starting time. This leads to the "process" formulation of a non-autonomous dynamical systems. It has the main advantage of being intuitively obvious, but a disadvantage is that it gives little insight into the underlying which cause the changes. An alternative formulation uses "skew-product flows", which consist of an autonomous dynamical system for the driving mechanism or noise and a cocycle mapping for the state space dynamics. A cocycle is a generalization of a semi-group to include information about the actual state of the driving system.

There are parallel theories for deterministic non-autonomous dynamical systems and random dynamical systems, with the essential difference being that topological arguments can be used in the first case while measure theoretic arguments are need in the second. These various formulations will be discussed, compared and illustrated with examples based on differential and difference equations.