Coupling multimodeling with local mesh refinement

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Resumen

We propose a twofold adaptive method based on a posteriori control of discretization error and modeling error with respect to functional output $j(u)$, see [1]. Denoting by u the continuous solution of a partial differential equation in variational formulation and by u_h the discrete solution of a discrete equation. The two formulations differ not only by the variational spaces but also with respect to different models entering the partial differential equation. The discrete variational formulation is considered to involve a simpler model. The a posteriori error representation derived in [3] is of the following form:

$$j(u) - j(u_h) \approx \eta_h + \eta_m + R,$$

where the terms η_h and η_m are the error estimators of the discretization error and the modeling error, respectively. The part η_h consists of residuals with respect to the simpler model and involves approximations of the interpolation error of the primal solution u and the interpolation error of an associated dual solution z. The modeling error estimator η_m involves the residual with respect to the more accurate model locally. As a consequence, the model changes from cell to cell in the computational domain.

The methodology is applied to combustion problems, were complicated diffusion models (multicomponent diffusion) are known but rarely used in practice, see [2] and [4], due to the high numerically cost. Therefore, we use also a simpler diffusion model (Fick's law) and measure the introduced error. In the adaptive process, we switch dynamically to the more accurate model (equation) and refine the mesh simultaneously.

Referencias

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