Parameter identification and applications in ultrasonic bio-imaging

JÉRÔME FEHRENBACH

Laboratoire MIP, Univ. Paul Sabatier - Toulouse (Francia)

fehren@mip.ups-tlse.fr

Resumen

Identification of some parameter of a partial differential equation (boundary conditions and/or coefficients) from the observation of solutions of this equation is a mathematically challenging task. Techniques issued from optimal control of partial differential equations can provide answers. Parameter identification has many interesting applications, we will present examples in ultrasonic bio-imaging.

The first example is the identification of elasto-static parameters of a material. The clinical interest is that tumours are harder than the surrounding safe tissue. We will present the imaging modality, named ultrasonic elastography. Under small displacements, biological tissues are assumed to be linear elastic materials. An observation is the measurement of one component of the displacement under an external loading. From this displacement data, the objective is to retrieve the stiffness of the material (Young's modulus).

The operator to be inverted is compact, therefore the identification of Young's modulus is an ill-posed problem. We propose an identification strategy that consists of minimizing the difference between the observation and the predictions using a Gauss-Newton algorithm. This optimization does not require the full Jacobian (it is expensive to compute and to store). The product of a vector by the Jacobian and by its transpose are obtained by direct and adjoint differentiation [1]. Examples on *in-vitro* data are presented. Poisson's ratio is a parameter describing the compressibility of a linear isotropic elastic medium. The influence of Poisson's ratio on the direct and on the inverse problems are discussed [2].

Ultrasound images can be obtained at a very high frame rate (∼5000 Hz) using emerging techniques [3]. This allows to image the shear waves propagating in a biological tissue (shear wave speed: 5-50 $m.s^{-1}$). The shear wave speed is related to the shear modulus and provides clinical information. Another information is of clinical interest: the attenuation coefficient. In principle, the shear modulus and the attenuation coefficient could be determined from the measurements. The methods currenty in use involve two differentiations of the measurements, or do not take into account the wave equation.

Other criteria can help discriminate between normal and pathological tissues. For instance a non-linear stress-strain relation, and anisotropic elasticity tensors characterize pathological tissues [4]. The mathematicians could help designing methods that estimate quantitatively these two behaviors. This would be interesting both on the mathematical point of view and for the practial applications.

Referencias

[1] J.Fehrenbach, M.Masmoudi, R.Souchon, P.Trompette *Detection of small inclusions using elastography*, Inverse Problems, **22** (2006), 1055–1069

[2] J Fehrenbach, *Influence of Poisson's ratio on elastographic direct and inverse problems*, Phys Med Biol, **52** (2007), 707–716

[3] M Fink, L Sandrin, M Tanter, S Catheline, S Chaffai, J Bercoff, JK Gennisson *2D transient elastography with an Ultrafast ultrasonic scanner* 1st Intl Conf Ultras. Meas. and Imag Tiss Elast (2002)

[4] T Krouskop, T Wheeler, F Kallel, *The elastic moduli of breast and prostate tissues under compression*, Ultrason Imaging, **20** (1998) 151–159