How to approximate the heat equation with Neumann boundary conditions by nonlocal diffusion problems

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Resumen

The purpose of this talk is to show that the solutions of the usual Neumann boundary value problem for the heat equation can be approximated by solutions of a sequence of nonlocal "Neumann" boundary value problems.

Let $J : \mathbf{R}^N \to \mathbf{R}$ be a nonnegative, radial, continuous function with $\int_{\mathbf{R}^N} J(z)\,dz = 1$. Assume also that J is strictly positive in $B(0,d)$ and vanishes in $\mathbf{R}^N \setminus B(0,d)$. Nonlocal evolution equations of the form $u_t(x,t) = (J*u-u)(x,t) = \int_{\mathbf{R}^N} J(x-y)u(y,t)\,dy - u(x,t)$, and variations of it, have been recently widely used to model diffusion processes, see [1], [2], [5].

In this talk, following [3] and [4], we propose a nonlocal "Neumann" boundary value problem, namely

$$u_t(x,t) = \int_\Omega J(x-y)\big(u(y,t)-u(x,t)\big)\,dy + \int_{\mathbf{R}^N\setminus\Omega} G(x,x-y)g(y,t)\,dy,$$

where $G(x,\xi)$ is smooth and compactly supported in ξ uniformly in x.

Now, for given J and G we consider the rescaled kernels

$$J_\varepsilon(\xi) = C_1 \frac{1}{\varepsilon^N} J\left(\frac{\xi}{\varepsilon}\right), \qquad G_\varepsilon(x,\xi) = C_1 \frac{1}{\varepsilon^N} G\left(x, \frac{\xi}{\varepsilon}\right)$$

and then the solution $u^\varepsilon(x,t)$ to

$$\begin{cases} u_t^\varepsilon(x,t) &= \dfrac{1}{\varepsilon^2}\displaystyle\int_\Omega J_\varepsilon(x-y)(u^\varepsilon(y,t)-u^\varepsilon(x,t))\,dy + \frac{1}{\varepsilon}\int_{\mathbf{R}^N\setminus\Omega} G_\varepsilon(x,x-y)g(y,t)\,dy, \\ u^\varepsilon(x,0) &= u_0(x). \end{cases}$$

We show that

$$u^\varepsilon \to u,$$

in different topologies according to different choices of the kernel G. Here u is the solution of the heat equation, $u_t = \Delta u$ with boundary condition $\partial u/\partial\eta = g$ and initial condition u_0.

This is a joint work with C. Cortazar, M. Elgueta and N. Wolanski.

Referencias

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