SESIÓN 3
SISTEMAS DINÁMICOS NO AUTÓNOMOS Y ESTOCÁSTICOS

ORGANIZADOR: Tomás Caraballo

Dpto. E.D.A.N., Univ. de Sevilla

caraball@us.es

Introduction to the theory of non-autonomous and stochastic/random dynamical systems

Peter E. Kloeden

Univ. JWG, Frankfurt (Alemania)

kloeden@math.uni-frankfurt.de

Resumen

The theory of autonomous dynamical systems and their attractors is well established. Essentially, the mechanism driving the dynamics does not change in time, and the dynamics can be described by a group or semi-group of mappings taking the state space into itself and attractors are compact sets which are invariant, i.e. mapped onto themselves, and attract all nearby trajectories. A particular characteristic of autonomous systems is that only the elapsed time since starting is important, not the actual and starting times themselves.

In non-autonomous dynamical systems and random dynamical systems the driving mechanism itself changes in time, which must also be built into the system description. The most obvious way to do this is to describe the dynamics by a two-parameter group of mappings of the state space into itself, with the parameters being the actual time and the starting time. This leads to the "process" formulation of a non-autonomous dynamical systems. It has the main advantage of being intuitively obvious, but a disadvantage is that it gives little insight into the underlying which cause the changes. An alternative formulation uses "skew-product flows", which consist of an autonomous dynamical system for the driving mechanism or noise and a cocycle mapping for the state space dynamics. A cocycle is a generalization of a semi-group to include information about the actual state of the driving system.

There are parallel theories for deterministic non-autonomous dynamical systems and random dynamical systems, with the essential difference being that topological arguments can be used in the first case while measure theoretic arguments are need in the second. These various formulations will be discussed, compared and illustrated with examples based on differential and difference equations.

Estado actual y problemas abiertos de la teoría de sistemas dinámicos no autónomos y/o estocásticos

José A. Langa

Dpto. E.D.A.N., Univ. de Sevilla

langa@us.es

Resumen

Desde hace ya casi dos décadas existe una importante investigación sobre las propiedades cualitativas de ecuaciones diferenciales bajo la presencia de términos no autómos y estocásticos. Se trata de modelos que, en general, permiten una aproximación a veces muy realista de fenómenos reales que provienen de otras ramas del saber científico. En estas condiciones, los correspondientes sistemas dinámicos gozan de unos grados de libertad tan grandes que el comportamiento asintótico de los mismos es a veces sorprendente y muy alejado de lo conocido en la Teoría Clásica de Sistemas Dinámicos. Sin embargo, los intensos estudios realizados por diversos grupos en distintas partes del mundo, sobre todo en la última década, permiten hoy describir un mapa de la situación en el que algunos de los principales problemas abiertos han quedado resueltos, permitiendo de esta manera que podamos hoy hablar de un cuerpo teórico coherente e independiente en la Teoría de Sistemas Dinámicos.

El objetivo de esta presentación es describir algunos de los resultados principales de esta teoría, indicando su novedad e importancia en relación con resultados anteriores, así como plantear algunos de los problemas abiertos más relevantes que están siendo actualmente analizados.

Structure and continuity properties of attractors for non-autonomous dynamical systems

James C. Robinson

Institute of Mathematics, Univ. de Warwick, Inglaterra

jcr@maths.warwick.ac.uk

Resumen

Although the theory of attractors for autonomous dynamical systems is well-developed, classically there is only one class of equations for which one has a good understanding of the structure of the attractor, namely gradient systems. In this case the attractor is the union of the unstable manifolds of the equilibria. Such attractors, whatever underlying system gives rise to them, can be shown to change continuously under perturbation, even when the perturbation is non-autonomous.

In addition, we show that the attractor of systems that are small non-autonomous perturbations of gradient systems have the same structure, giving the first class of non-autonomous systems in which we have a good understanding of the structure of the attractor.