SESIÓN 4
CONTROL AND INVERSE PROBLEMS FOR PARTIAL DIFFERENTIAL EQUATIONS

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A 0-Laplacian approach to impedance imaging

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Resumen

Electrical impedance tomography (EIT) technique has been an active research topic since the early 1980s. In EIT, one measures the boundary voltages due to multiple injection currents to reconstruct images of the conductivity distribution. However, these boundary voltages are insensitive to a local change of the conductivity distribution and the relation between them is highly nonlinear.

Medical imaging has been one of the important application areas of EIT. Indeed, biological tissues have different electrical properties that change with cell concentration, cellular structure, and molecular composition. Such changes of electrical properties are the manifestations of structural, functional, metabolic, and pathological conditions of tissues, and thus provide valuable diagnostic information. Since all the present EIT technologies are only practically applicable in feature extraction of anomalies, improving EIT calls for innovative measurement techniques that incorporate structural information.

The core idea of the approach presented in this talk is to extract more information about the conductivity from data that has been enriched by coupling the electric measurements to localized elastic perturbations. More precisely, we propose to perturb the medium during the electric measurements, by focusing ultrasonic waves on regions of small diameter inside the body. Using a simple model for the mechanical effects of the ultrasound waves, we show that the difference between the measurements in the unperturbed and perturbed configurations is asymptotically equal to the pointwise value of the energy density at the center of the perturbed zone. In practice, the ultrasounds impact a spherical or ellipsoidal zone, of a few millimeters in diameter. The perturbation should thus be sensitive to conductivity variations at the millimeter scale, which is the precision required for breast cancer diagnostic.

The material presented in this talk conerning the imaging by perturbation approach, is based on a joint work with Habib Ammari, Eric Bonnetier, Michael Tanter & Matthias Fink and on an ongoing collaboration with Frdric de Gournay, Otared Kavian and Jrme Fehrenbach. I will also discuss recent results concerning perturbation of asymptotically small volume fraction which are based on joint works with Michael Vogelius.

Parameter identification and applications in ultrasonic bio-imaging

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Resumen

Identification of some parameter of a partial differential equation (boundary conditions and/or coefficients) from the observation of solutions of this equation is a mathematically challenging task. Techniques issued from optimal control of partial differential equations can provide answers. Parameter identification has many interesting applications, we will present examples in ultrasonic bio-imaging.

The first example is the identification of elasto-static parameters of a material. The clinical interest is that tumours are harder than the surrounding safe tissue. We will present the imaging modality, named ultrasonic elastography. Under small displacements, biological tissues are assumed to be linear elastic materials. An observation is the measurement of one component of the displacement under an external loading. From this displacement data, the objective is to retrieve the stiffness of the material (Young's modulus).

The operator to be inverted is compact, therefore the identification of Young's modulus is an ill-posed problem. We propose an identification strategy that consists of minimizing the difference between the observation and the predictions using a Gauss-Newton algorithm. This optimization does not require the full Jacobian (it is expensive to compute and to store). The product of a vector by the Jacobian and by its transpose are obtained by direct and adjoint differentiation [1]. Examples on *in-vitro* data are presented. Poisson's ratio is a parameter describing the compressibility of a linear isotropic elastic medium. The influence of Poisson's ratio on the direct and on the inverse problems are discussed [2].

Ultrasound images can be obtained at a very high frame rate (∼5000 Hz) using emerging techniques [3]. This allows to image the shear waves propagating in a biological tissue (shear wave speed: 5-50 $\mathrm{m.s}^{-1}$). The shear wave speed is related to the shear modulus and provides clinical information. Another information is of clinical interest: the attenuation coefficient. In principle, the shear modulus and the attenuation coefficient could be determined from the measurements. The methods currenty in use involve two differentiations of the measurements, or do not take into account the wave equation.

Other criteria can help discriminate between normal and pathological tissues. For instance a non-linear stress-strain relation, and anisotropic elasticity tensors characterize pathological tissues [4]. The mathematicians could help designing methods that estimate quantitatively these two behaviors. This would be interesting both on the mathematical point of view and for the practial applications.

Referencias

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Cloaking: a new phenomena in Electromagnetism and Elasticity

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Resumen

The making of an object invisible through some cloaking device until recently was commonly regarded as science fiction. Two quite different types of electromagnetic cloaking were proposed in early 2006. In our cloaking scenario a collection of finitely many polarizable dipoles becomes essentially invisible when they are within a certain critical distance of a superlens.

Superlenses have attracted attention because they promise resolution on a length scale finer than can be achieved using conventional lenses, i.e. finer than the wavelength. The radiation scattered by the polarizable dipoles resonates with the superlens and acts back on the dipoles to essentially cancel the field incident on them, which is why they become invisible. Dipolar energy sources supplying constant power also become invisible. A second type of cloaking was proposed by Pendry, Schurig and Smith and Leonhardt. In this scenario a shield cloaks objects to incident electromagnetic waves by guiding the waves around the object.

This work is related to the earlier work of Greenleaf, Lassas and Uhlmann, on cloaking for conductivity. Here we will review these developments and also discuss how cloaking might be extended to elasticity us- ing these ideas. This requires new materials, in particular materials with anisotropic mass density and a constitutive law in which the stress depends on the velocity and the momentum depends on the displacement gradient. We sketch how such materials, with behavior outside that of continuum elastodynamics, might be made. This is joint work with Lindsay Botten, Marc Briane, Ross McPhedran, Nicolae Nicorovici, and John Willis.