SESIÓN 7
APPROXIMATION THEORY AND SPECIAL FUNCTIONS WITH APPLICATIONS

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Rational spectral transformations and orthogonal polynomials

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Resumen

$k-$Toeplitz matrices are tridiagonal matrices of the form $A = [a_{i,j}]_{i,j=1}^{n}$ (with $n \geq k$) such that $a_{i+k,j+k} = a_{i,j}$, $(i, j = 1, 2, \cdots, n-k)$, so that they are $k-$periodic along the diagonals parallel to the main diagonal. When $k = 1$ it reduces to a tridiagonal Toeplitz matrix. The interest of the study of $k-$Toeplitz matrices appears to be very important not only from a theoretical point of view (in linear algebra or numerical analysis, e.g.), but also in applications. Here in this talk, motivated by certain physical systems (namely a system of quantum oscillators with a nonlinear interactions) we will discuss spectral properties of some tridiagonal quasi-periodic as well as certain perturbations of them.

Transformadas de Dunkl y teoremas de muestreo

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Resumen

Sea $\alpha \geq -1/2$ (aunque muchas cosas se pueden extender hasta $\alpha > -1$). Para funciones adecuadas, la transformada de Dunkl sobre la recta real se define como

$$\mathcal{F}_\alpha(f, y) = \int_{\mathbf{R}} E_\alpha(-ixy) f(x)\, d\mu_\alpha(x), \qquad y \in \mathbf{R},$$

donde $d\mu_\alpha$ es la medida

$$d\mu_\alpha(x) = \frac{1}{2^{\alpha+1}\Gamma(\alpha+1)} |x|^{2\alpha+1}\, dx$$

y E_α denota cierta función que se expresa en términos de las funciones de Bessel. Cuando $\alpha = -1/2$, $E_{-1/2}(z) = e^z$ y $\mathcal{F}_{-1/2}$ es la transformada de Fourier.

El primero que usó la transformada que ahora se denomina de Dunkl fue Roosenraad en su tesis doctoral [6], escrita bajo la dirección de Richard Askey, aunque aparentemente pasó desapercibida. Pero, desde que Dunkl [4] la reintrodujo en 1989, muchos investigadores se han ocupado de estudiar sus propiedades, intentado adaptar a un contexto más amplio todo tipo de resultados ya conocidos sobre la transformada de Fourier. Véanse, por ejemplo, los recientes artículos [1, 2, 5, 7, 8, 9].

Aquí, siguiendo [3], presentamos un teorema de muestreo relacionado con la transformada de Dunkl. En el caso $\alpha = -1/2$, dicho teorema se reduce al teorema de Whittaker-Shannon-Kotel'nikov clásico.

(En colaboración con Oscar Ciaurri.)

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Asymptotic methods for convolution integrals unified and demystified

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Resumen

We present a new method for deriving asymptotic expansions of $\int_0^\infty f(t)h(xt)dt$ for small x. We only require for f(t) and h(t) to have asymptotic expansions at $t = \infty$ and $t = 0$ respectively. Remarkably, it is a very general technique that unifies a certain set of asymptotic methods. Watson's Lemma and other classical methods, Mellin transform techniques, McClure and Wong's distributional approach and the method of analytic continuation turn out to be simple corollaries of this method. In addition, the most amazing thing about it is that its mathematics are absolutely elemental and do not involve complicated analytical tools as the aforesaid methods do: it consists of simple "sums and substractions". Many known and unknown asymptotic expansions of important integral transforms are trivially derived from the approach presented here.