SESIÓN 9
GOAL ORIENTED ADAPTIVE METHODS FOR THE NUMERICAL SOLUTION OF PDES

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Coupling multimodeling with local mesh refinement

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Resumen

We propose a twofold adaptive method based on a posteriori control of discretization error and modeling error with respect to functional output $j(u)$, see [1]. Denoting by u the continuous solution of a partial differential equation in variational formulation and by u_h the discrete solution of a discrete equation. The two formulations differ not only by the variational spaces but also with respect to different models entering the partial differential equation. The discrete variational formulation is considered to involve a simpler model. The a posteriori error representation derived in [3] is of the following form:

$$j(u) - j(u_h) \approx \eta_h + \eta_m + R,$$

where the terms η_h and η_m are the error estimators of the discretization error and the modeling error, respectively. The part η_h consists of residuals with respect to the simpler model and involves approximations of the interpolation error of the primal solution u and the interpolation error of an associated dual solution z. The modeling error estimator η_m involves the residual with respect to the more accurate model locally. As a consequence, the model changes from cell to cell in the computational domain.

The methodology is applied to combustion problems, were complicated diffusion models (multicomponent diffusion) are known but rarely used in practice, see [2] and [4], due to the high numerically cost. Therefore, we use also a simpler diffusion model (Fick's law) and measure the introduced error. In the adaptive process, we switch dynamically to the more accurate model (equation) and refine the mesh simultaneously.

Referencias

[1] R. Becker and R. Rannacher. *An optimal control approach to a posteriori error estimation in finite element methods.* In A. Iserles, editor, Acta Numerica 2001. Cambrige University Press, 2001.

[2] M.Braack and A. Ern. *Coupling multimodeling with local mesh refinement for the numerical solution of laminar flames.* Submitted to Combustion Theory and Modeling, 2003.

[3] M. Braack and A. Ern. *A posteriori control of modeling errors and discretization errors.* Multiscale Model. Simul., 1(2): 221-238, 2003.

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Adaptive space-time finite element methods for parabolic optimization problems

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Resumen

In this talk we discuss a posteriori error estimates for space-time finite element discretization of parabolic optimization problems. The provided error estimates assess the discretization error with respect to a given quantity of interest and separate the influence of different parts of the discretization (time, space and control discretization). This allows to set up an efficient adaptive algorithm which successively improves the accuracy of the computed solution by construction of locally refined meshes for time and space discretizations.

A time-space adaptive semi-DWR method

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Resumen

We present in this talk a time-space adaptive semi-DWR method in the framework of finite elements. We use goal oriented adaptation of a functional $J(u)$ of the solution to estimate the error based on the Dual Weighted Residual methodology. The main ingredients of our new time-space adaptive method are: (1) Use of both structured and unstructured meshes. (2) Estimation of the local truncation error by solving backward at each time step a dual problem in the subinterval $[t_{n-1}, t_n]$; so that we avoid to solving backward the dual problem in the whole time integration interval $[0, T]$. (3) The local error estimators for the goal functional $J(u)$ yield a very effective adaptive algorithm which allows to having a control on both the size of the time step and the size of the mesh elements.

We shall illustrate the capabilities of our method when it is applied to solve several reaction-diffusion-convection problems as well as the Navier-Stokes equations, in which the convection terms are treated in a semi-Lagrangian manner.