The mathematics of markets: existence, uniqueness and stability of competitive equilibria.

Antonio Villar

Universidad Pablo de Olavide.

This conference deals with the mathematical tools that permits one to understand the logics of competitive markets. The problem was originally formulated in a precise way by Len Walras in the XIXth Century but could only be solved around 1954. It referred to the solvability of a finite system of equations with non-negativity restrictions. Each equation corresponds to the equality between supply and demand in a particular market, prices being the adjustment variables. The existence of a solution relies on the application of a fixpoint theorem. The interpretation is that there exists a price vector that is capable of equalizing supply and demand in all markets simultaneously. Once the existence of a solution is ensured, new questions appear immediately. Is the solution unique? Do the price dynamics converge to the equilibrium price vector? A condition concerning the responsiveness of the equations with respect to changes in the variables (quite close to the condition defining Z-functions), does the job. Some changes in the environment will also be considered: non-competitive behaviour, non-convex feasible sets, non-finite sets of markets, a continuum of agents, etc.