A new look at compactness via distances to function spaces

Bernardo Cascales

Universidad de Murcia.

In this lecture we will show that several classical results about compactness in functional analysis can all be derived from some suitable inequalities involving distances to spaces of continuous or Baire one functions: this gives an extra insight to the classical results. We will exhibit *quantitative* versions of Grothendieck's characterization of weak compactness in spaces $C(K)$ and also for the Eberlein-Grothendieck and Krein-Smulyan theorems. The above results are specialized in Banach spaces: we obtain several equivalent measures of *non-weak compactness*. In a different direction we envisage a method to measure the distance from a function $f \in \mathbb{R}^X$ to $B_1(X)$ –space of Baire one functions on X– which allows us to obtain, when X is Polish, a quantitative version of the well known Rosenthal's result stating that in $B_1(X)$ the pointwise relatively countably compact sets are pointwise compact. We also obtain quantitative versions of a result by Srivatsa and another result by Namioka about the existence of points of joint continuity for a separately continuous functions.