The Banach space $L_p[0,1]$

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We will discuss the Banach space structure of $L_p[0,1]$ mostly in the reflexive setting, $1 < p < \infty$. This classical Banach space has been a prime case study for abstraction to a more general study of Banach space structure. It has an unconditional basis, namely the Haar basis. The Rademacher functions span a subspace isomorphic to Hilbert space, and complemented in L_p. There are uncountably many nonisomorphic complemented subspaces and all of these, except Hilbert space, exhibit a local structure like that of L_p. If $p > 2$ and $q \neq p$ ($q \neq 2$) then L_q does not embed into L_p. For $p < 2$, L_q embeds into L_p iff $p \leq q \leq 2$. For $1 < p < \infty$ every normalized unconditional sequence in L_p lies between the ℓ_p and ℓ_2 norms. Every subspace (infinite dimensional) of L_p ($p > 2$) either contains ℓ_p or is isomorphic to ℓ_2. If such a subspace does not contain ℓ_2 it embeds into ℓ_p. The norm on L_p is stable and as such does not distort ℓ_p.

These talks will go slowly and treat some of the above in depth while only touching on others.