Problems on hypercyclic operators

Héctor Salas

Universidad of Puerto Rico, Mayagüez.

A linear operator T acting on a Fréchet space E is hypercyclic if there exists $x \in E$ such that the $\mathrm{Orb}(T, x) = \{T^n(x) : n = 0, 1, 2, , ...\}$ is dense in E; this implies that E must be infinite dimensional and separable. Although instances of this phenomenon have been known for many years, its study didn't begin in earnest until around twenty years ago. Since then many people have contributed to developing and understanding of the structure of these operators. There are hypercyclic operators which are weighted shifts and others that are composition operators, but no hypercyclic operator can be either compact or hyponormal (on a Hilbert space).

An operator T is dual hypercyclic if both T and its adjoint T^* are hypercyclic. This concept was recently studied by H. Petersson. We will explain why every infinite dimensional separable complex Banach space with separable dual supports dual hypercyclic operators. There are examples of these operators whose spectra are disks or annuli containing the unit circle. There are also examples whose spectra are disks centered at λ with $|\lambda| = 1$. A well-known result of S. Ansari which says that powers of hypercyclic operators are also hypercyclic allows us to conclude that T^n is dual hypercyclic for $n = 1, 2, 3, \ldots$ in the case T is. Assume further that $\sigma(T) = \{z : |z-1| \leq \frac{1}{2}\}$. Then by the spectral theorem $\sigma(T^n) = \{z^n : |z - 1| \leq \frac{1}{2}\}$. It is also known that no dual hypercyclic operator can be chaotic; i.e, can have a denumerable set of periodic points. Very recently, A. Montes-Rodrguez, A. Rodrguez-Martnez, and S. Shkarin have found new classes of dual hypercyclic operators, which shed a new light on the subject and give us greater insight into these operators.