An introduction to local discrete holomorphic dynamics

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A local discrete holomorphic dynamical system at a point p in a complex manifold M is a holomorphic map $f: U \to M$ with f(p) = p, where U is a (usually small) open neighbourhood of p in M. A natural set associated to such a system is the stable set $K_f \subseteq U$ of points whose orbit is completely contained in U, where the orbit of a point $z \in U$ is the set $\{f^k(z)\}$, where $f^k = f \circ \cdots \circ f$ is the k-th iterate of f. The aim of the theory of local discrete dynamical systems is to study the topology of the stable set, and the dynamics of f on the stable set. One of the main tools for such a study is to replace f by another, possibly simpler, map locally conjugated to fnearby p; thus this opens the natural question of classifying local dynamical systems under topological, holomorphic or formal conjugation.

In this short course we shall summarize the theory in complex dimension 1, describing what is known on the topological and dynamical structure of the stable set, and on topological, holomorphic and formal conjugacy classes.