Some results on the reliability of multiple path-shaped facility location on networks

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Abstract

In this paper we study a location problem on networks that combines three important issues: 1) it considers that facilities are extensive, 2) it handles simultaneously the location of more than one facility, and 3) it incorporates reliability aspects related to the fact that facilities may fail. The problem consists of locating two path-shaped facilities minimizing the expected service cost in the long run, assuming that paths may become unavailable and their failure probabilities are known in advance. We discuss several aspects of the computational complexity of problems of locating two or more reliable paths on graphs, showing that multifacility path location - with and without reliability issues - is a difficult problem even for 2 facilities and on very special classes of graphs. In view of this, we focus on trees and provide a polynomial time algorithm that solves the 2 unreliable path location problem on tree networks in $O(n^2)$ time, where n is the number of vertices.

Keywords:

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1 Introduction

One of the most important strategic decision in the design of infrastructures is the location of facilities. This has motivated a lot of research on different facility location problems during many years (see, e.g., [7]) and, in particular, on several extensive path- or tree-shaped facility location problems (see, e.g., [14,15,17] and the references therein).

There are several sources of uncertainty that must be considered when facing a location problem. Costs, customer demands or production capacities may be unknown at the moment of making a decision, but also unexpected events may disrupt the workability of the facilities themselves. This gives rise to situations where some facilities become temporary unavailable to provide service to customers, due to system failures, natural disasters, terrorist attacks, labor strikes, etc.. Therefore, modeling issues should handle as best as they can these unknown or unpredictable situations whenever they occur.

Realistically, no decision-maker would accept a solution with very high operating costs just to hedge against very rare facility disruptions, unless high penalties must be paid to costumers in case of uncovered service. Failures typically result in extra transportation costs as customers originally served by the closest facilities must be redirected to more distant ones (see, e.g., [8]). In order to balance the normal and failure operation costs, the facility location should depend on how likely facilities may get disrupted, as well as, on their closeness to the potential customers. This has motivated an alternative approach to the "customer-to-closest facility cost" criterion that consists of locating facilities that minimize the total expected service cost in the long run, assuming that failures are accidental, and their probabilities can be estimated in advance [3,16]. Needless to say, this approach is not unique and another way to model disruption is to consider it motivated by intentional attacks

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[4,12,21]. Nevertheless, although these models are very interesting, they are beyond the scope of this paper.

The literature in the area of reliable location models can be traced back, at least, to the paper [6] where both a median and a center objective functions are considered, and the situation where a fixed number of facilities might fail is tackled. However, one can observe that the current interest in reliability issues in location has been recently restarted with [19] and other works by the same authors. In addition, the following papers [3], [22], [5] and [18] have also contributed decisively to this increasing interest. Robustness analysis of transportation networks has been also widely studied in the literature from alternative points of view (see, i.e., [1,11,13,20]).

As can be seen from the literature review, there are models that consider reliability aspects of point location and some other models that apply integer programming tools to design routes, but one does not find reliability models for the simultaneous location of extensive facilities. The goal of this paper is to combine three crucial aspects of location models: (i) the existence of more than one service facility; (ii) the assumption of the extensive nature of service facilities (frequently more realistic than the assumption of locating points); (iii) the minimization of the total expected service cost in the long run, assuming that failures are accidental and probability of failure are known. In particular, referring to (i) and (ii), we point out that there are few papers in the literature that consider the problem of locating two or more path-shaped facilities on networks. The combination of the three above aspects makes the location problem studied in this paper even more difficult under the computational complexity viewpoint, also for graphs with a very simple structure. We discuss several aspects related to the computational complexity of problems that are strictly related to our unreliable path location problem and, in fact, are special cases of it. We show that multiple facility path location problems are NP-Hard even on very simple classes of graphs, implying that the same holds for our reliability problem. This suggests that there is little hope to solve even the two unreliable path location problem on networks more general than a tree.

In view of the above considerations, we focus on trees and study the following problem: given two path-shaped facilities that may fail with given probabilities, find two paths in the tree where the two facilities can be located in order to minimize the total expected service cost. Here we assume that each customer is first assigned to its closest facility, then, if this fails, to the second closest, and, if both facilities fail, he/she is assigned to a backup facility modeled by a penalty cost. Assuming that the tree has n vertices, the problem can be solved by brute force by evaluating the objective function on each of the $O(n^4)$ different pairs of paths of the tree. Since each evaluation can be done in linear time, this approach would lead to an overall complexity of $O(n^5)$ time. In spite of that, in this paper we present an $O(n^2)$ time complexity algorithm for solving this problem. It is also worth noticing that our complexity result for locating two paths on a tree equals the one obtained by [3] for the corresponding two points location problem.

Although the multiple unreliable path location problem studied in this paper can be considered as a natural extension of the point location version already analyzed in [3], some additional issues arise when handling pairs of paths instead of pairs of points. As we will see, this requires new algorithmic solutions, in particular for an efficient evaluation of pairs of *intersecting* paths.

The paper is organized as follows. In Section 2 we provide some notation and definitions, while basic properties are introduced in Section 3. In Section 4 we discuss the computational complexity of the problem of locating $K \geq 1$ paths on networks also in the case of unreliable paths. Section 5 illustrates the algorithm for solving the two path-shaped facilities location problem with probabilities of failure. In Section 6 we discuss further extensions and draw some conclusions.

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